A Comparison between the Performance of Number-State and Coherent-State Optical CDMA in Lossy Photon Channels

Hossam M. H. Shalaby, *Member, IEEE*

Abstract—**The performance of optical code-division multiple-access (CDMA) communication systems utilizing number-state light field is evaluated. Lossy direct-detection optical channels are assumed. Both on-off keying (OOK) and pulse-position modulation (PPM) schemes are investigated. For OOK, the exact bit error rate is evaluated taking into account the effect of both multiple-user interference and transmission loss. Upper and lower bounds on the bit error probability for PPM-CDMA systems are derived under the above considerations. The effect of both background and thermal noise is neglected in our analysis. Performance comparison between the number-state and coherent-state OOK/PPM-CDMA networks is also presented. Our results demonstrate that the number-state system requires less than half the energy consumed by the coherent-state one in order to attain the same performance. Lower bounds on the maximum number of simultaneous users are derived for both number- and coherent-state PPM-CDMA systems with asymptotically zero error rate.**

I. Introduction

THE quantum fluctuations of light photons generated by a *coherent state* laser lead to an uncertainty in estimating the number of photons contained in a coherent light pulse. This number can be modeled as a Poisson random variable with parameter (or mean) equals the average photon count per pulse [22]. A coherent state is a minimum uncertainty state that satisfies the Heisenberg uncertainty principle between two canonical physical quantities of light. The uncertainty relation between the photon number n and the phase ϕ is

$$\Delta n^2 \cdot \Delta\phi^2 \geq \tfrac{1}{4}$$

where Δx denotes the fluctuation (uncertainty or noise) of quantity x. It is not possible, in the coherent state, to reduce either of the above fluctuations below certain threshold. In other words, coherent state exhibits uncontrolled sources of noise. On the other hand, new quantum states of light (squeezed state and sub-Poisson state) whose two quantum noises can be controlled have been studied [1]–[7]. In these quantum states one source of noise can be reduced at the expense of increasing the other so as to maintain the uncertainty relation unchanged. In the sub-Poisson state it is possible to reduce the photon number fluctuation below that of the coherent state by increasing the uncertainty of the phase. In the limit where we have no (or zero) photon fluctuation, we get what is called photon number state. Thus the photon count contained in a light pulse generated by a *number state* laser is a nonrandom unique value. That is, every transmitted photon, in a lossless channel, will appear as is at the receiving end. This property is useful in optical communication systems utilizing direct detection. If the optical channel is lossy, however, some of the transmitted photons may disappear before reaching the photodetector. Assuming that $\eta \in [0, 1]$ denotes the transmittance coefficient of the lossy channel, the probability of detecting exactly n photons given that m photons have been transmitted can be written as

$$\Pr\{n|m\} = \binom{m}{n}\eta^n(1-\eta)^{m-n}, \qquad n \in \{0, 1, \cdots, m\}.$$

The above equation demonstrates that number state optical pulses also yield random photon counting processes at the receiving end if the channel is lossy. Therefore performance degradation is expected as η decreases even in the absence of the background noise.

Recently, an increasing interest has been given to optical code-division multiple-access (CDMA) techniques because of their ultrafast signal-processing speeds [8]–[18]. Several models for optical CDMA communication networks have been suggested in literature. In a typical system model there are N simultaneous sources of information (users) which produce continuous and asynchronous data. The data of each user modulates a laser source using either on-off keying (OOK) or M-ary pulse-position modulation (PPM) schemes. Each modulated signal is then multiplied by a periodic signature (code) sequence of length L and weight w. Denoting the bit rate by R_0 b/s, the chip time T_c of the sequence can be shown to be given by

$$T_c = \begin{cases} \dfrac{1}{LR_0}, & \text{for OOK} \\ \dfrac{\log_2 M}{MLR_0}, & \text{for PPM} \end{cases} \tag{1}$$

where M denotes the number of possible pulse positions (slots) within a PPM time frame. Instead of the above multiplication process, we can equivalently use an optical encoder which is composed of an optical splitter, tapped optical delay lines, and an optical combiner [8], [21]. The output optical pulses

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The author is with the Department of Electrical Engineering, Faculty of Engineering, University of Alexandria, Alexandria 21544, Egypt.

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of each multiplier (or optical encoder) undergo transmission loss in the channel before reaching the receiver. The received waveform is composed of the sum of N delayed and attenuated signals from each user in addition to the background noise. Each user performs its own decoding technique by multiplying the received waveform by the same underlying code sequence and integrating over the desired interval. Also instead of the multiplication operation we can use an optical matched filter (or correlator) characterized by the specific code sequence of the desired user [11], [12], [15], [18]. Finally the output of the correlator is forwarded to an OOK/PPM demodulator which decides on the true data.

In this paper we aim at comparing between the bit error rate performance of coherent and number state optical CDMA systems utilizing either OOK or PPM modulation techniques in a direct-detection optical channel. In our theoretical analysis we consider the effect of the transmission loss due to the attenuation in the optical channel. We neglect, however, the effect of both the background and thermal noise. In order to have some insight on the results obtained we assume chip-synchronous uniformly-distributed relative delays among the transmitters and perfect photon counting processes at the receivers.

In the numerical analysis, we employ optical orthogonal codes (OOC's) [19], [20] as the signature code sequences. To have minimal interference between the users we adopt OOC's with periodic cross-correlations and out-of-phase periodic autocorrelations that are bounded only by 1 [19].

The remaining of our paper is organized as follows. Section II is devoted for the derivation of the bit error rate for optical OOK-CDMA through both coherent and number state lossy channels. Upper and lower bounds on the bit error probability for PPM-CDMA systems are derived in Section III. Performance comparisons between the coherent and number state channels are illustrated at the end of the above two sections. In Section IV we derive asymptotically ($M \to \infty$) achievable expressions for the maximum number of simultaneous users that can be accommodated by both number- and coherent-state PPM-CDMA systems. Finally extensions and concluding remarks are given in Section V.

II. Bit Error Rate for OOK-CDMA

In OOK a signature sequence is transmitted (of w laser pulses) to represent data bit "1". Data bit "0" is represented, however, by zero pulses. We denote by κ the number of pulses (from the other users) that cause interference to the desired user. In OOC's with cross-correlations bounded by one, each undesired user may contribute only one pulse to this number or contribute no pulses at all (since we assume chip synchronous). Hence κ is a binomial random variable with parameters $w^2/2L$ and $N-1$ [17]

$$\Pr\{\kappa = l\} = \binom{N-1}{l}\left(\frac{w^2}{2L}\right)^l\left(1-\frac{w^2}{2L}\right)^{N-1-l}, \quad l \in \{0, 1, \cdots, N-1\}. \tag{2}$$

The Decision Rule: As usual, a threshold θ is set. If the collected photon count in one bit time is less than this threshold, "0" is declared, otherwise "1" is declared to be sent. The probability of bit error is thus given by

$$\begin{aligned} P_b(\theta) &= \tfrac{1}{2}(P[E|0] + P[E|1]) \\ &= \tfrac{1}{2}\sum_{l=0}^{N-1}(P[E|0, \kappa = l] \\ &\quad + P[E|1, \kappa = l])\Pr\{\kappa = l\} \end{aligned} \tag{3}$$

where $P[E|i,\ \kappa = l]$ is the probability of error given that $i \in \{0,\ 1\}$ was sent and there are l interfering pulses with the desired user. To evaluate this probability of error we consider the following two cases (A and B).

A. Number State

We assume that exactly m photons are contained in each transmitted pulse, i.e., a total of m photons are transmitted during the bit time of data bit "1". A decoding error can thus occur (given that "0" has been sent and l pulses have interfered with the desired user) if the number of collected photons is at least equal to θ

$$P[E|0, \kappa = l] = \begin{cases} \displaystyle\sum_{n=\theta}^{ml}\binom{ml}{n}\eta^n(1-\eta)^{ml-n}, & \text{if } l \geq \dfrac{\theta}{m} \\ 0, & \text{otherwise.} \end{cases} \tag{4a}$$

Similarly

$$P[E|1, \kappa = l] = \begin{cases} \displaystyle\sum_{n=0}^{\theta-1}\binom{m(w+l)}{n}\eta^n(1-\eta)^{m(w+l)-n}, & \text{if } l > \dfrac{\theta-1}{m} - w \\ 1, & \text{otherwise.} \end{cases} \tag{4b}$$

B. Coherent State

Assuming that the average transmitted photons per pulse equals m, then the average received photons per pulse (due to channel loss) becomes ηm. Hence for a PIN photodetector which output is a Poisson random variable [22]

$$\begin{aligned} P[E|0, \kappa = l] &= \sum_{n=\theta}^{\infty}\exp[-\eta m l]\frac{(\eta m l)^n}{n!} \\ P[E|1, \kappa = l] &= \sum_{n=0}^{\theta-1}\exp[-\eta m(w+l)] \\ &\quad \cdot \frac{(\eta m(w+l))^n}{n!}. \end{aligned} \tag{5}$$

Numerical Results: The optimum threshold which minimizes the bit error rate in (3) has been evaluated numerically for $w = 5$, $L = 500$, and different values of η, N, and m. The minimum bit error rate $P_b = \min_\theta P_b(\theta)$ is plotted in Figs. 1 and 2 for both number state and coherent state. The superiority of the number state network is obvious from the figures. For example if $N = 5$, $\eta = 0.7$, and $P_b \leq 10^{-5}$ we need (at

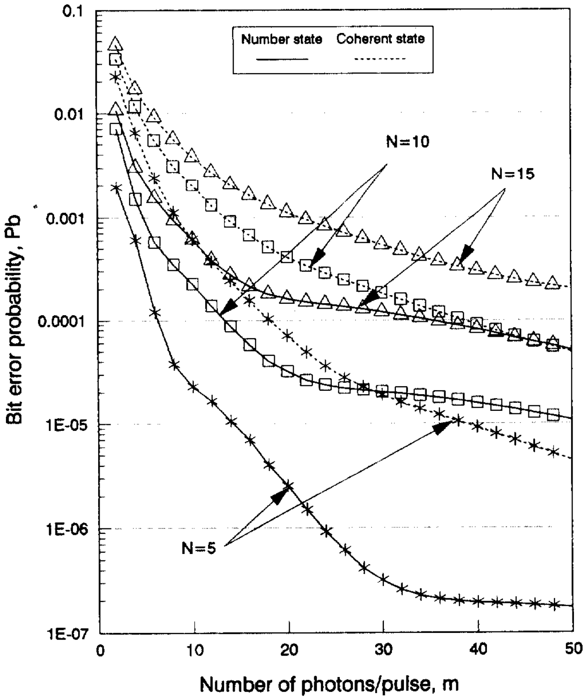

Fig. 1. Bit error probability as a function of the number of users and photons/pulse for OOK-CDMA with $\eta = 0.7$, $w = 5$, and $L = 500$.

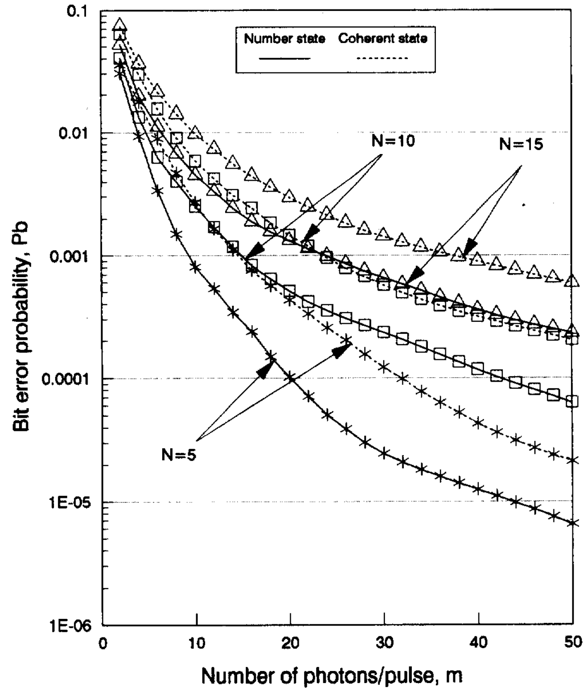

Fig. 2. Bit error probability as a function of the number of users and photons/pulse for OOK-CDMA with $\eta = 0.4$, $w = 5$, and $L = 500$.

least) $m = 15$ for the number state whereas $m = 40$ for the coherent state. This indicates that more than 60% save in energy is gained when using the number state OOK. It is also noticed that for $\eta = 0.7$ the performance of the number state system with ten simultaneous users is almost similar to the coherent state system with only five simultaneous users. Also for $\eta = 0.4$ or 0.7, the performance of the number state system with 15 users is competitive to the coherent one with ten users.

III. Bit Error Rate for PPM-CDMA

In M-ary PPM a time frame of duration T is divided into M disjoint slots each having a width $\tau = T/M$. Symbol $i \in \{0, 1, \cdots, M-1\}$ is represented by transmitting a signature sequence within slot number i. We denote by κ_i, $i \in \{0, 1, \cdots, M-1\}$ the number of pulses (from other users) that cause interference to slot i of the desired user. As in the case of OOK, κ_i is a binomial random variable but with parameters $N-1$ and w^2/ML. The joint distribution of any two random variables κ_i and κ_j, $i \neq j$ is given by

$$\begin{aligned}\Pr\{\kappa_i = l_i, \kappa_j = l_j\} &= \sum_{r=0 \vee l_i + l_j - (N-1)}^{l_i \wedge l_j} \cdot \frac{(N-1)!}{r!(l_i - r)!(l_j - r)!(N-1-l_i-l_j+r)!} \\ &\cdot P_{11}^r(i,j) P_{10}^{l_i - r}(i,j) P_{01}^{l_j - r}(i,j) \\ &\cdot P_{00}^{N-1-l_i-l_j+r}(i,j), \\ &l_i, l_j \in \{0, 1, \cdots, N-1\} \quad (6)\end{aligned}$$

where $x \wedge y \stackrel{\text{def}}{=} \min\{x, y\}$, $x \vee y \stackrel{\text{def}}{=} \max\{x, y\}$, and

$$\begin{aligned}P_{11}(i,j) &= \frac{|i-j|}{M^2} \cdot \frac{w^2}{ML} \\ P_{10}(i,j) &= \left(1 - \frac{|i-j|}{M^2}\right) \frac{w^2}{ML} \\ P_{01}(i,j) &= P_{10}(i,j) \\ P_{00}(i,j) &= 1 - P_{11}(i,j) - P_{10}(i,j) - P_{01}(i,j). \quad (7)\end{aligned}$$

The derivation of this distribution can be found in Appendix A.

1) The Decision Rule: We denote the photon count collected in slot $i \in \{0, 1, \cdots, M-1\}$ by Y_i. Symbol "i" is declared to be transmitted if $Y_i > Y_j$ for every $j \neq i$; an incorrect decision is made if the photon counts for two or more slots are the same. We now provide a union bound on the probability of word error P_E. The bit error rate P_b is related to P_E by the well-known formula $P_b = (M/2/(M-1))P_E$

$$P_E = \sum_{i=0}^{M-1} P[E|i] \Pr\{i\}$$

where $\Pr\{i\} = 1/M$ in the case of equally likely data and

$$\begin{aligned}P[E|i] = \Pr\{Y_j \geq Y_i, \text{ some } j \neq i | i\} \\ \leq \sum_{j=0, j \neq i}^{M-1} \Pr\{Y_j \geq Y_i | i\}.\end{aligned}$$

Denoting the union bound by P_E^U and noticing that $\Pr\{Y_j \geq Y_i | i\}$ depends only on the absolute value of the difference

$|i-j|$, we get

$$P_E^U = \frac{1}{M}\sum_{i=0}^{M-1}\sum_{j=0,j\neq i}^{M-1} \Pr\{Y_j \geq Y_i|i\}$$
$$= \frac{2}{M}\sum_{d=1}^{M-1}(M-d)\Pr\{Y_j \geq Y_i|i, |i-j|=d\}$$
$$= \frac{2}{M}\sum_{d=1}^{M-1}(M-d)\Pr\{Y_d \geq Y_0|0\}. \tag{8}$$

The probability under the summation can be evaluated as follows

$$\Pr\{Y_d \geq Y_0|0\} = \sum_{l_0=0}^{N-1}\sum_{l_d=0}^{N-1} \cdot \Pr\{Y_d \geq Y_0|0, \kappa_0 = l_0, \kappa_d = l_d\} \cdot \Pr\{\kappa_0 = l_0, \kappa_d = l_d|0\}$$
$$= \sum_{l_0=0}^{N-1}\sum_{l_d=0}^{N-1} \cdot \Pr\{Y_d \geq Y_0|0, \kappa_0 = l_0, \kappa_d = l_d\} \cdot \Pr\{\kappa_0 = l_0, \kappa_d = l_d\}. \tag{9}$$

This union bound is still too complex, however. We thus provide tight upper and lower bounds on the above union bound.

2) Lower Bound on P_E^U: We can write

$$\Pr\{Y_d \geq Y_0|0\} \geq \sum_{l_d=0}^{N-1}\Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} \cdot \Pr\{\kappa_0 = 0, \kappa_d = l_d\}.$$

Using (6) and (7), we obtain

$$\Pr\{\kappa_0 = 0, \kappa_d = l_d\} = \binom{N-1}{l_d} P_{01}^{l_d}(0,d) P_{00}^{N-1-l_d}(0,d)$$

where

$$P_{01}(0,d) = \left(1 - \frac{d}{M^2}\right)\frac{w^2}{ML}$$
$$P_{00}(0,d) = 1 - \frac{w^2}{ML} - P_{01}(0,d).$$

Hence

$$P_E^U \geq \frac{2}{M}\sum_{l_d=0}^{N-1}\Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} \cdot \sum_{d=1}^{M-1}(M-d)\binom{N-1}{l_d} \cdot P_{01}^{l_d}(0,d) P_{00}^{N-1-l_d}(0,d). \tag{10}$$

3) Upper Bound on P_E^U: This bound is provided by noticing that

$$\Pr\{Y_d \geq Y_0|0, \kappa_0 = l_0, \kappa_d = l_d\} \leq \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\}.$$

Hence by substitution in (9)

$$\Pr\{Y_d \geq Y_0|0\} \leq \sum_{l_0=0}^{N-1}\sum_{l_d=0}^{N-1} \cdot \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} \cdot \Pr\{\kappa_0 = l_0, \kappa_d = l_d\}$$
$$= \sum_{l_d=0}^{N-1}\Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} \cdot \Pr\{\kappa_d = l_d\}.$$

Using (8), we obtain

$$P_E^U \leq \frac{2}{M}\sum_{l_d=0}^{N-1} \cdot \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} \cdot \sum_{d=1}^{M-1}(M-d)\binom{N-1}{l_d}\left(\frac{w^2}{ML}\right)^{l_d} \cdot \left(1 - \frac{w^2}{ML}\right)^{N-1-l_d}$$
$$= (M-1)\sum_{l_d=0}^{N-1}\binom{N-1}{l_d}\left(\frac{w^2}{ML}\right)^{l_d} \cdot \left(1 - \frac{w^2}{ML}\right)^{N-1-l_d} \cdot \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\}. \tag{11}$$

What remains to complete the evaluation of the error bounds is to get expressions on $\Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\}$ under both number state (A) and coherent state (B) assumptions.

A. Number State

Assuming that exactly m photons are transmitted per pulse, we can write

$$\Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} = \sum_{n_d=0}^{ml_d}\binom{ml_d}{n_d}\eta^{n_d}(1-\eta)^{ml_d - n_d} \times \sum_{n_0=0}^{n_d \wedge mw}\binom{mw}{n_0}\eta^{n_0}(1-\eta)^{mw-n_0}. \tag{12}$$

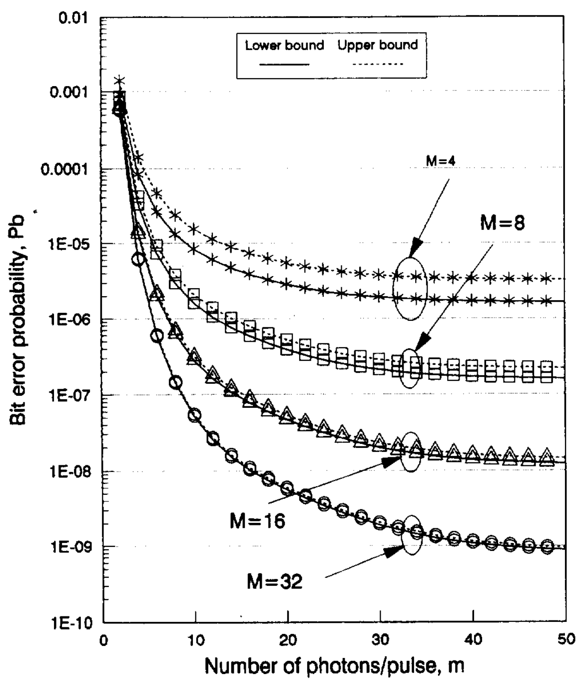

Fig. 3. Upper and lower bounds on the bit error rate union bound as a function of the pulse position multiplicity and photons/pulse for number-state PPM-CDMA with $\eta = 0.7$, $w = 5$, $L = 500$, and $N = 20$.

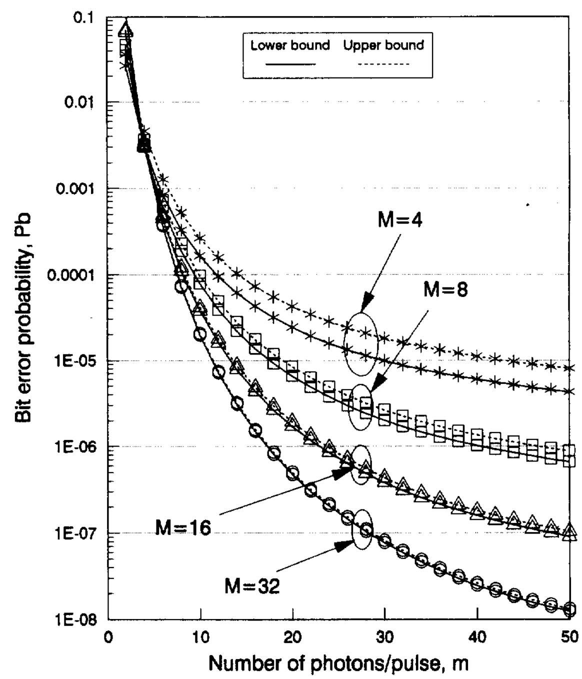

Fig. 4. Upper and lower bounds on the bit error rate union bound as a function of the pulse position multiplicity and photons/pulse for number-state PPM-CDMA with $\eta = 0.4$, $w = 5$, $L = 500$, and $N = 20$.

B. Coherent State

Assuming that the average transmitted photons per pulse is equal to m, we have

$$\Pr\{Y_d \geq Y_0 | 0, \kappa_0 = 0, \kappa_d = l_d\} = \sum_{n_d=0}^{\infty} \exp[\eta m l_d] \frac{(\eta m l_d)^{n_d}}{n_d!} \cdot \sum_{n_0=0}^{n_d} \exp[\eta m w] \frac{(\eta m w)^{n_0}}{n_0!}. \quad (13)$$

Numerical Results: Upper and lower bounds on P_E^U have been evaluated numerically for the case of number state with $w = 5$, $L = 500$, $N = 20$, and different values of η, M, m. These bounds (scaled to the bit error probability) are shown in Figs. 3 and 4. It is clear that the upper bound on P_E^U is so close (same order of magnitude) to the true union bound especially for large M. Because of its simplicity we use the upper bound on P_E^U in the following numerical analysis. A comparison between number- and coherent-state bit error rate is shown in Figs. 5 and 6 under the above parameter values. The superiority of the number state system over the coherent state one is clear from the figures. As an example, if $N = 20$, $\eta = 0.7$, and $P_b \leq 10^{-7}$, at least $m = 9$ photons/pulse are required for the number state whereas $m = 28$ for the coherent state if $M = 32$. When $M = 16$, m becomes 16 in the case of the number state and becomes 48 for the coherent state. The above numbers indicate that energy is saved by more than 66% when using the number state PPM. Another remark on the curves is that the performance improves as M increases. From Fig. 5 with $P_b \leq 10^{-7}$ there is about 44% save in energy per pulse when switching the number state system from $M = 16$ to $M = 32$. This percentage is, however, misleading; a fair comparison should be based on the transmitted photons per bit not per pulse. Hence for $M = 32$, $m * w / \log_2 M = 9*5/5 = 9$ photons/b is consumed versus $16 * 5/4 = 20$ photons/b for $M = 16$ to attain the above bit error rate. That is, the true save in energy is about 55% (not 44%). A serious problem in system realization may arise as M increases above 32 because the chip time must be decreased in order to hold the bit rate fixed. The resulting laser pulsewidth will in turn be too difficult to generate with the current optical technology. A quick look at the curves suggests a crucial solution to the above problem by using number state systems instead of the coherent state. The performance of the number state with $M = 16(8)$ is almost competitive to the coherent state system with $M = 32(16)$ for m exceeding 30.

IV. Lower Bounds to the Maximum Number of Simultaneous Users

PPM is one of the most efficient techniques that can be used over ideal optical direct-detection channels [23]–[25]. In [26] we have derived an achievable expression for the maximum number of users ($N_{M,\max}$) that can communicate simultaneously with asymptotically zero error rate in a *synchronous* coherent optical PPM-CDMA system with lossless channels.

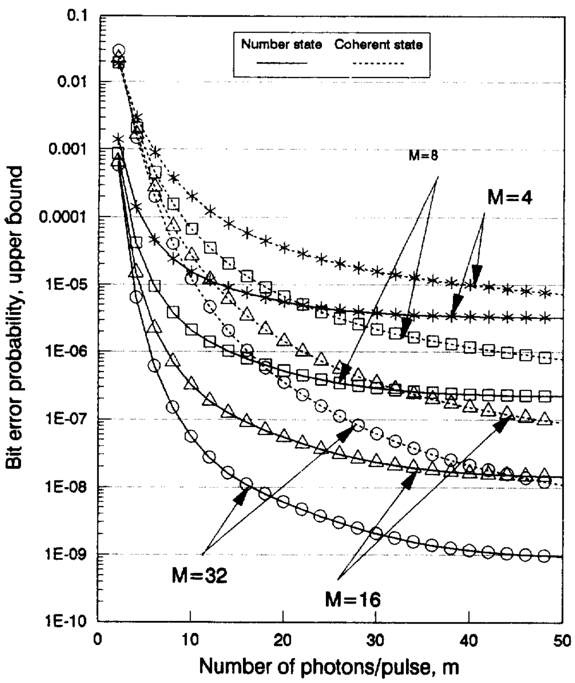

Fig. 5. Bit error probability upper bound as a function of the pulse position multiplicity and photons/pulse for PPM-CDMA with $\eta = 0.7$, $w = 5$, $L = 500$, and $N = 20$.

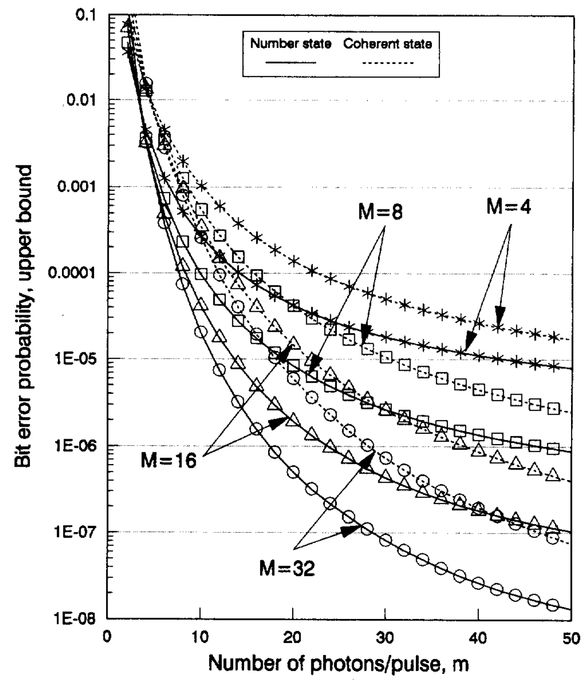

Fig. 6. Bit error probability upper bound as a function of the pulse position multiplicity and photons/pulse for PPM-CDMA with $\eta = 0.4$, $w = 5$, $L = 500$, and $N = 20$.

Namely, we have shown that

$$\liminf_{M\to\infty} \frac{N_{M,\max} - 1}{M^{(1+\rho w - e^{\rho})/\rho w}} \geq we^{-(1+\rho)}.$$

In other words, for any $\delta > 0$ arbitrary small if $N < 1 + (1 - \delta)we^{-(1+\rho)}M^{(1+\rho w - e^{\rho})/\rho w}$, then $\lim_{M\to\infty} P_E = 0$. Here ρ denotes the transmitted information in nats/photon

$$\rho \overset{\text{def}}{=} \frac{\log M}{mw}.$$

In the case of number state, m denotes the exact number of photons transmitted per pulse, hence m is an integer. In the case of coherent state m denotes the average photons per transmitted pulse.

In this section we aim at deriving similar expressions for both chip-synchronous number- and coherent-state optical PPM-CDMA systems with lossy channels. The intended expressions are provided in the following two theorems.

Theorem 1: For optical orthogonal code sequences with length L, weight w, and cross-correlations bounded by one, the maximum number of simultaneous users, in an optical number-state chip-synchronous PPM-CDMA system, is lower bounded by

$$\liminf_{M\to\infty} \frac{N_{M,\max} - 1}{M^{[\rho w - \log(1-\eta+\eta e^{\rho/\eta})/\rho w]}} \geq \frac{L}{w}(\eta + (1-\eta)e^{-\rho/\eta})e^{-1/[\eta+(1-\eta)e^{-\rho/\eta}]}$$

or

$$\liminf_{M\to\infty} \frac{\log(N_{M,\max} - 1)}{\log M} \geq 1 - \frac{1}{\rho w}\log(1 - \eta + \eta e^{\rho/\eta})$$

where ρ, η, and M denote the transmitted information in nats per photon, the transmittance coefficient, and the pulse position multiplicity, respectively.

Proof: An upper bound on P_E (based on Chernoff inequality) is given in Appendix B. We start by estimating suitable values of $s \in [0, 1]$ and $z \geq 1$ for this upper bound so as to have asymptotically zero error rate. It suffices to show that the expression in the right-hand side of (B1) is positive. We denote this expression by $sh(s)$

$$h(s) \overset{\text{def}}{=} -\rho - \frac{1}{s}\log(1 - \eta + \eta z^{-s}) - \frac{N-1}{\log M/\rho} \cdot \log\left[1 - \frac{w^2}{ML} + \frac{w^2}{ML}(1 - \eta + \eta z)^{\log M/\rho w}\right].$$

From the continuity of $h(s)$ for every $s \in [0,1]$, we have

$$h(s) = \lim_{s\to 0} h(s) - o(s)$$

where $o(s) \to 0$ as $s \to 0$. Whence it suffices to show that $\lim_{s\to 0} h(s) > 0$. Indeed, this implies that for $s > 0$ small enough $h(s) \geq \delta$, where $\delta > 0$ arbitrary small which in turn leads to $P_E \leq \exp[-s\delta \log M/\rho]$. Hence P_E decreases to zero as $M \to \infty$. It is easy to check that

$$\lim_{s\to 0} h(s) = \eta \log z - \rho - \frac{N-1}{\log M/\rho} \log\left[1 - \frac{w^2}{ML} + \frac{w^2}{ML} \cdot (1-\eta+\eta z)^{\log M/\rho w}\right]$$
$$\geq \eta \log z - \rho - \frac{N-1}{\log M/\rho} \frac{w^2}{ML}(1-\eta+\eta z)^{\log M/\rho w}$$
$$= \eta \log z - \rho - \frac{(N-1)\rho w^2}{L \log M} \cdot M^{[\log(1-\eta+\eta z)-\rho w]/\rho w}.$$

The last term is positive if

$$N - 1 < \frac{\eta \log z - \rho}{M^{[\log(1-\eta+\eta z)-\rho w]/\rho w} \dfrac{\rho w^2}{L \log M}}, \qquad z \geq 1.$$

Hence

$$N_{M,\max} - 1 \geq \max_{z\geq 1} \frac{\eta \log z - \rho}{M^{[\log(1-\eta+\eta z)-\rho w]/\rho w} \dfrac{\rho w^2}{L \log M}}.$$

To obtain the maximum of the expression in the right-hand side we differentiate it with respect to z and equate the result with zero. Hence the optimizing z^* must satisfy

$$(z^*)^{-1} - (\eta \log z^* - \rho) \frac{\log M/\rho w}{1 - \eta + \eta z^*} = 0.$$

From Appendix C we conclude that

$$\exp\left[\frac{\rho}{\eta}\right] \leq z^* \leq \exp\left[\frac{\rho}{\eta}\left(1 + \frac{w}{\log M}\right)\right].$$

Whence

$$N_{M,\max} - 1 \geq \frac{\eta \log z^* - \rho}{M^{[\log(1-\eta+\eta z^*)-\rho w]/\rho w} \dfrac{\rho w^2}{L \log M}}$$
$$= \frac{L}{w} \cdot \frac{\eta + (1-\eta)(z^*)^{-1}}{M^{\log(1-\eta+\eta z^*)-\rho w]/\rho w}}$$
$$\geq \frac{L}{w} \frac{\eta + (1-\eta)\exp\left[-\dfrac{\rho}{\eta}\left(1 + \dfrac{w}{\log M}\right)\right]}{M^{\{\log(1-\eta+\eta \exp[(\rho/\eta)(1+[w/\log M])])-\rho w\}/\rho w}}. \tag{14}$$

Taking the limit as $M \to \infty$, we obtain

$$\liminf_{M\to\infty} \frac{N_{M,\max} - 1}{M^{[\rho w - \log(1-\eta+\eta e^{\rho/\eta})]/\rho w}} \geq \frac{L}{w}(\eta + (1-\eta)e^{-\rho/\eta})e^{-1/[\eta+(1-\eta)e^{-\rho/\eta}]}.$$

□

Theorem 2: For optical orthogonal code sequences with length L, weight w, and cross-correlations bounded by one, the maximum number of simultaneous users, in an optical coherent-state chip-synchronous PPM-CDMA system, is lower bounded by

$$\liminf_{M\to\infty} \frac{N_{M,\max} - 1}{M^{(\rho w + \eta - \eta e^{\rho/\eta})/\rho w)}} \geq \frac{L}{w} e^{-(1+\rho/\eta)}$$

or

$$\liminf_{M\to\infty} \frac{\log(N_{M,\max} - 1)}{\log M} \geq 1 + \frac{1}{\rho w}(\eta - \eta e^{\rho/\eta})$$

where ρ, η, and M denote the transmitted information in nats per photon, the transmittance coefficient, and the pulse position multiplicity, respectively.

Proof: The proof is similar to that of Theorem 1 with slight modifications. □

It is obvious from the previous two theorems that the estimate of the lower bound on the maximum number of users in a number state is greater than that of a coherent state. These estimates become close to each other when $\rho \to 0$.

In the case of lossless channel, $\eta = 1$, the lower bounds given by the above two theorems reduce to (cf. (14) above)

$$N_{M,\max} \geq \begin{cases} 1 + \dfrac{L}{w} e^{-1} M^{(w-1)/w}, & \text{for number state} \\ 1 + (1-\delta_M)\dfrac{L}{w} e^{-(1+\rho)} M^{(\rho w + 1 - e^{\rho})/\rho w}, & \text{for coherent state} \end{cases}$$

where $\delta_M \to 0$ as $M \to \infty$. The last expression for the number state does not depend on ρ, which is an expected result if the channel is lossless where every transmitted photon will appear at the receiving end. It can also be noticed that the lower bound for the number state users increases with M. Since the maximum number of subscribers cannot exceed $N \leq (L-1)/w(w-1)$ [19], there always exists an $M^* > 0$ such that if $M \geq M^*$, all the subscribers can communicate simultaneously with acceptable performance. The last conclusion can also be withdrawn for the case of coherent sate but under the condition

$$\rho w + 1 > e^{\rho}.$$

If this condition is not satisfied then the lower bound decreases as M increases.

V. Extensions and Concluding Remarks

Bit error rates for optical chip-synchronous CDMA communication systems utilizing both number- and coherent-state light fields have been derived for lossy direct-detection photon channels. Exact expressions have been obtained for the case of an OOK modulation scheme. Tight upper and lower bounds on the union bound have been provided when PPM is used. The effect of the multiple-user interference and transmission loss has been considered in the numerical analysis. Our results suggest using the number state system instead of the coherent state one in optical CDMA because of its superiority over the latter. Namely, the number state system requires less than half the energy consumed by the coherent state one to

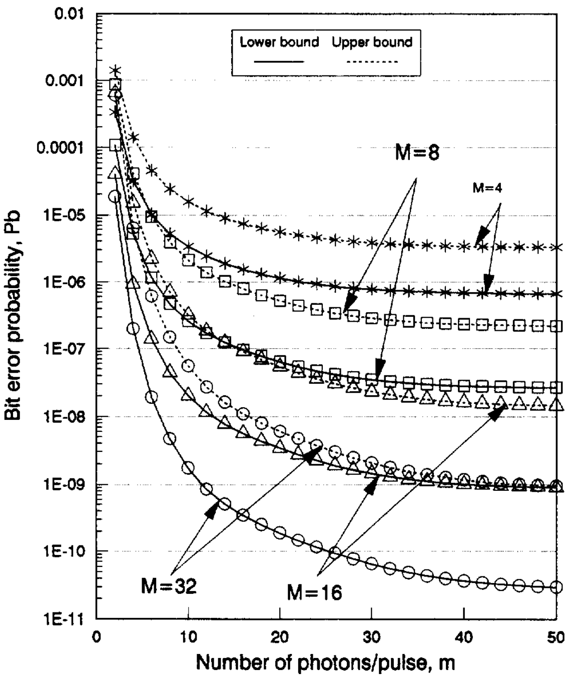

Fig. 7. Upper and lower bounds on the bit error probability as a function of the pulse position multiplicity and photons/pulse for number-state PPM-CDMA with $\eta = 0.7$, $w = 5$, $L = 500$, and $N = 20$.

attain the same performance. That is the maximum number of simultaneous users is larger in the case of number state.

In our analysis of PPM-CDMA we have used an upper bound on the bit error rate. In order to figure out the uncertainty on the exact P_b, we provide the following lower bound

$$\begin{aligned}
P_E &= \sum_{i=0}^{M-1} P[E|i] \Pr\{i\} \\
&= \sum_{i=0}^{M-1} \Pr\{i\} \Pr\{Y_j \geq Y_i, \text{ some } j \neq i|i\} \\
&\geq \sum_{i=0}^{M-2} \Pr\{i\} \Pr\{Y_{i+1} \geq Y_i|i\} \\
&\quad + \Pr\{M-1\} \Pr\{Y_{M-2} \geq Y_{M-1}|M-1\} \\
&= \Pr\{Y_1 \geq Y_0|0\} \\
&= \sum_{l_0=0}^{N-1} \sum_{l_1=0}^{N-1} \Pr\{Y_1 \geq Y_0|0, \kappa_0 = l_0, \kappa_1 = l_1\} \\
&\quad \cdot \Pr\{\kappa_0 = l_0, \kappa_1 = l_1\} \\
&\geq \sum_{l_1=0}^{N-1} \Pr\{Y_1 \geq Y_0|0, \kappa_0 = 0, \kappa_1 = l_1\} \\
&\quad \cdot \Pr\{\kappa_0 = 0, \kappa_1 = l_1\}.
\end{aligned}$$

The upper and lower bounds on P_b have been evaluated numerically for the case of number state with $w = 5$, $L = 500$, $N = 20$, $\eta = 0.7$, and different values of M, m. These bounds are shown in Fig. 7. It is clear that the upper bound determines the exact bit error rate within 1.5 orders of magnitude.

Theorems 1 and 2 are incomplete in the sense that they provide only lower bounds on the maximum achievable number of users. In the meantime we are trying to find converse results by providing tight upper bounds.

Appendix A

First we assume that we have only one interfering user. The probability that this single user causes exactly one interference pulse (hit) in slot i of the desired user is given by $P_i(1) = w^2/ML$ [16]. It is easy see that the probability of exactly one interference pulse (hit) in slot j given that a hit has occurred in slot i, $j \neq i$, is given by

$$P_{j|i}(1|1) = \frac{|i-j|}{M^2}.$$

The probability of exactly one hit in slot j given that no hits have occurred in slot i, $j \neq i$, is thus given by

$$\begin{aligned}
P_{j|i}(1|0) &= \frac{P_{i,j}(0,1)}{P_i(0)} = \frac{P_j(1) - P_{i,j}(1,1)}{P_i(0)} \\
&= \frac{P_j(1) - P_i(1)P_{j|i}(1|1)}{P_i(0)} \\
&= \frac{w^2/ML}{1 - w^2/ML}\left(1 - \frac{|i-j|}{M^2}\right).
\end{aligned}$$

Now we assume that there are $N-1$ interfering users. Hence the probability of l_j hits in slot j given l_i hits have occurred in slot i, $j \neq i$, is calculated as follows. Out of the l_j hits in slot j there are r hits due to those users who contribute hits in both slots i and j. The remaining $l_j - r$ hits are due to those users who contribute hits in only slot j and not in i. The number r, thus, can not exceed l_i and l_j. On the other hand, the value $l_j - r$ can not exceed $N-1-l_i$, i.e., $r \geq l_j - (N-1-l_i)$. Whence

$$\begin{aligned}
&\Pr\{\kappa_j = l_j|\kappa_i = l_i\} \\
&= \sum_{r=0 \vee l_j-(N-1-l_i)}^{l_i \wedge l_j} \binom{l_i}{r} P_{j|i}^r(1|1)(1 - P_{j|i}(1|1))^{l_i - r} \\
&\quad \cdot \binom{N-1-l_i}{l_j - r} P_{j|i}^{l_j - r}(1|0) \\
&\quad \cdot (1 - P_{j|i}(1|0))^{N-1-l_i-(l_j-r)}.
\end{aligned}$$

□

Appendix B

We show that for any two real numbers $s \in [0,1]$ and $z \geq 1$, the probability of a word error, P_E, in the optical number-state chip-synchronous PPM-CDMA channel can be upper bounded as

$$\begin{aligned}
-\frac{\log P_E}{\log M/\rho} &\geq -s\rho - \log(1 - \eta + \eta z^{-s}) \\
&\quad - s\frac{N-1}{\log M/\rho} \log\left[1 - \frac{w^2}{ML} + \frac{w^2}{ML}(1 - \eta + \eta z)^{\log M/\rho w}\right].
\end{aligned} \tag{B1}$$

Proof: The probability of correct decision can be written as

$$\begin{aligned} P_C &= \sum_{i=0}^{M-1} P[C|i] \Pr\{i\} \\ &= \sum_{i=0}^{M-1} \Pr\{i\} \sum_{l_0^{M-1}} P[C|i, \kappa_0^{M-1} = l_0^{M-1}] \\ &\quad \cdot \Pr\{\kappa_0^{M-1} = l_0^{M-1}\}. \end{aligned}$$

where l_0^{M-1} and κ_0^{M-1} denote the vectors $(l_0, \cdots, l_{M-1})$ and $(\kappa_0, \cdots, \kappa_{M-1})$, respectively, and

$$\begin{aligned} P[C|i, \kappa_0^{M-1} = l_0^{M-1}] &\geq P[C|i, \kappa_0^{M-1} = l_0, \cdots, l_{i-1}, 0, l_{i+1}, \cdots, l_{M-1}] \\ &= P[Y_i > Y_j, j \neq i| \\ &\qquad i, \kappa_0^{M-1} = l_0, \cdots, l_{i-1}, 0, l_{i+1}, \cdots, l_{M-1}] \\ &= \sum_{n_i=0}^{mw} \binom{mw}{n_i} \eta^{n_i}(1-\eta)^{mw-n_i} \\ &\quad \cdot \prod_{j=0, j\neq i}^{M-1} \sum_{n_j=0}^{(n_i-1)\wedge ml_j} \binom{ml_j}{n_j} \\ &\quad \cdot \eta^{n_j}(1-\eta)^{ml_j-n_j}. \end{aligned}$$

For any $s \in [0, 1]$ the probability of decoding error given i and $\kappa_0^{M-1} = l_0^{M-1}$ can thus be bounded as (B2), shown at the bottom of this page, where we have made use of Appendix D. The second summation in the brackets can further be bounded as follows. For any $z \geq 1$

$$\begin{aligned} &\sum_{n_j=n_i}^{ml_j} \binom{ml_j}{n_j} \eta^{n_j}(1-\eta)^{ml_j-n_j} \\ &\qquad \leq \sum_{n_j=n_i}^{ml_j} z^{n_j-n_i} \binom{ml_j}{n_j} \eta^{n_j}(1-\eta)^{ml_j-n_j} \\ &\qquad \leq z^{-n_i} \sum_{n_j=0}^{ml_j} z^{n_j} \binom{ml_j}{n_j} \eta^{n_j}(1-\eta)^{ml_j-n_j} \\ &\qquad = z^{-n_i}(1-\eta+\eta z)^{ml_j}. \end{aligned}$$

Substituting in (B2) yields

$$\begin{aligned} &P[E|i, \kappa_0^{M-1} = l_0^{M-1}] \\ &\qquad \leq (1-\eta+\eta z^{-s})^{mw} \left[\sum_{j=0, j\neq i}^{M-1} (1-\eta+\eta z)^{ml_j} \right]^s. \end{aligned}$$

Hence the probability of decoding error given i can be obtained by the second equation at the bottom of this page. The convexity of the function x^s is utilized in the last inequality. Using the relation $P_E = \Sigma_{i=0}^{M-1} P[E|i] \Pr\{i\}$ yields

$$\begin{aligned} -\frac{\log P_E}{\log M/\rho} &\geq -s\rho - \frac{mw}{\log M/\rho} \log(1-\eta+\eta z^{-s}) \\ &\quad - s\frac{N-1}{\log M/\rho} \log\left[1 - \frac{w^2}{ML} + \frac{w^2}{ML}(1-\eta+\eta z)^m\right]. \end{aligned}$$

Substituting for $m = \log M/\rho w$ in the last expression completes the proof. □

$$\begin{aligned} P[E|i, \kappa_0^{M-1} = l_0^{M-1}] &\leq \sum_{n_i=0}^{mw} \binom{mw}{n_i} \eta^{n_i}(1-\eta)^{mw-n_i} \left[1 - \prod_{j=0, j\neq i}^{M-1} \left(1 - \sum_{n_j=n_i}^{ml_j} \binom{ml_j}{n_j} \eta^{n_j}(1-\eta)^{ml_j-n_j}\right)\right] \\ &\leq \sum_{n_i=0}^{mw} \binom{mw}{n_i} \eta^{n_i}(1-\eta)^{mw-n_i} \left[\sum_{j=0, j\neq i}^{M-1} \sum_{n_j=n_i}^{ml_j} \binom{ml_j}{n_j} \eta^{n_j}(1-\eta)^{ml_j-n_j}\right]^s \end{aligned} \quad \text{(B2)}$$

$$\begin{aligned} P[E|i] &= \sum_{l_0^{M-1}} P[E|i, \kappa_0^{M-1} = l_0^{M-1}] \Pr\{\kappa_0^{M-1} = l_0^{M-1}\} \\ &\leq (1-\eta+\eta z^{-s})^{mw} \sum_{l_0^{M-1}} \Pr\{\kappa_0^{M-1} = l_0^{M-1}\} \left[\sum_{j=0, j\neq i}^{M-1} (1-\eta+\eta z)^{ml_j}\right]^s \\ &\leq (1-\eta+\eta z^{-s})^{mw} \left[\sum_{j=0, j\neq i}^{M-1} \sum_{l_j=0}^{N-1} \Pr\{\kappa_j = l_j\}(1-\eta+\eta z)^{ml_j}\right]^s \\ &= (M-1)^s(1-\eta+\eta z^{-s})^{mw}\left[1 - \frac{w^2}{ML} + \frac{w^2}{ML}(1-\eta+\eta z)^m\right]^{(N-1)s} \end{aligned}$$

Appendix C

Consider the function

$$g(z) \stackrel{\text{def}}{=} \eta + (1-\eta)z^{-1} - (\eta \log z - \rho)\frac{\log M}{\rho w}, \qquad z \geq 1$$

where M and w are positive integers, $\rho > 0$, and $\eta \in [0,1]$. We show that the solution of the equation $g(z) = 0$ must satisfy the inequalities

$$\exp\left[\frac{\rho}{\eta}\right] \leq z \leq \exp\left[\frac{\rho}{\eta}\left(1 + \frac{w}{\log M}\right)\right].$$

Proof: This function is monotonically decreasing in z as long as $z \geq 1$. Indeed since the first derivative of $g(\cdot)$ is negative

$$(\forall z \geq 1) \qquad \frac{dg}{dz}(z) = -(1-\eta)z^{-2} - \eta z^{-1}\frac{\log M}{\rho w} < 0.$$

Hence it suffices to show that

$$g\left(\exp\left[\frac{\rho}{\eta}\right]\right) \geq 0$$

and

$$g\left(\exp\left[\frac{\rho}{\eta}\left(1 + \frac{w}{\log M}\right)\right]\right) \leq 0.$$

Indeed, we have

$$g\left(\exp\left[\frac{\rho}{\eta}\right]\right) = \eta + (1-\eta)e^{-\rho/\eta} \geq 0$$

and

$$\begin{aligned} g&\left(\exp\left[\frac{\rho}{\eta}\left(1 + \frac{w}{\log M}\right)\right]\right) \\ &= \eta + (1-\eta)\exp\left[-\frac{\rho}{\eta}\left(1 + \frac{w}{\log M}\right)\right] - 1 \\ &= -(1-\eta)\left\{1 - \exp\left[-\frac{\rho}{\eta}\left(1 + \frac{w}{\log M}\right)\right]\right\} \leq 0. \qquad \square \end{aligned}$$

Appendix D

Let $x_i \in [0,1], i \in \{1, 2, \cdots, M\}$, where $M \geq 1$ is any integer. We show that

$$1 - \prod_{i=1}^{M}(1 - x_i) \leq \left(\sum_{i=1}^{M} x_i\right)^s$$

for any $s \in [0,1]$.

Proof: First we show that

$$1 - \prod_{i=1}^{M}(1 - x_i) \leq \sum_{i=1}^{M} x_i. \qquad \text{(D1)}$$

Define the function

$$f(M) \stackrel{\text{def}}{=} \prod_{i=1}^{M}(1 - x_i) + \sum_{i=1}^{M} x_i.$$

It suffices to show that $f(M) \geq 1$ for any integer $M \geq 1$. We use the induction method in our proof.

1) True for $M = 1$:

$$f(1) = (1 - x_1) + x_1 = 1.$$

2) Assume True for $M = l$:

$$f(l) = \prod_{i=1}^{l}(1 - x_i) + \sum_{i=1}^{l} x_i \geq 1. \qquad \text{(D2)}$$

3) We Show That it is Also True for $M = l + 1$:

$$\begin{aligned} f(l+1) &= \prod_{i=1}^{l+1}(1 - x_i) + \sum_{i=1}^{l+1} x_i \\ &= (1 - x_{l+1})\prod_{i=1}^{l}(1 - x_i) + x_{l+1} + \sum_{i=1}^{l} x_i \\ &= f(l) + x_{l+1} - x_{l+1}\prod_{i=1}^{l}(1 - x_i) \geq 1 \end{aligned}$$

where we have made use of (D2) and the fact that $x_i \leq 1$. Now the left-hand side of (D1) is less than one. Whence for any $s \in [0,1]$

$$1 - \prod_{i=1}^{M}(1 - x_i) \leq \left(1 - \prod_{i=1}^{M}(1 - x_i)\right)^s \leq \left(\sum_{i=1}^{M} x_i\right)^s. \quad \square$$

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Hossam M. H. Shalaby (S'83–M'91) was born in Giza, Egypt, in 1961. He received the B.S. and M.S. degrees from the University of Alexandria, Egypt, in 1983 and 1986, respectively, and the Ph.D. degree from the University of Maryland, College Park, in 1991, all in electrical engineering.

He is currently an Assistant Professor with the Faculty of Engineering, University of Alexandria, Egypt. His research interests include communication theory, optical communications, signal detection and estimation, and information theory.

Dr. Shalaby was a recipient of an SRC fellowship (1987–1991).