Maximum Achievable Number of Users in Optical PPM-CDMA Local Area Networks

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***Abstract*—An optical code-division multiple-access (CDMA) communication network employing optical orthogonal codes is considered. The data symbols of each multiple-access user is encoded, before multiplexing, using pulse-position modulation (PPM) technique with sufficiently large pulse position multiplicity. The concepts of both users rate and users strength are introduced. Using these concepts an achievable number of simultaneous users that can be accommodated by the optical PPM-CDMA channel, while keeping the transmitted information per photon fixed and maintaining the probability of error below some prescribed threshold $0 < \epsilon < 1$, is determined. Furthermore, it is shown that the users strength has a simple positive characterization and in turn it is possible to load the entire subscribers simultaneously into the optical channel and embrace arbitrary small error rate.**

***Index Terms*—Code division multiple access (CDMA), direct detection optical channel, optical CDMA, pulse-position modulation (PPM), spread spectrum.**

I. Introduction

In OPTICAL code-division multiple-access (CDMA) systems, N users transmit information simultaneously over a common optical channel, Fig. 1. Each user is assigned a code (called the signature code) with length L and weight w. We focus on optical orthogonal codes (OOCs) with both off-peak autocorrelation and cross-correlation bounded by $\lambda \in \{1, 2, \ldots, w-1\}$. Methodologies in the design and analysis of such codes can be found in [1], [2]. An optical pulse (laser on) of duration T_c is transmitted whenever a "1" occurs in the signature code and a "0" is transmitted (laser off) for the same duration, otherwise.

Each user generates M-ary data symbols $D \in \{0, 1, \ldots, M-1\}$. These symbols are encoded with the aid of pulse-position modulation (PPM) schemes. In PPM, a time frame of duration T is subdivided into M disjoint slots, each slot has a width $\tau = T/M$. The user's information is conveyed by transmitting a signature code (with w optical pulses) in one of the M possible slots within the time frame. The relation between the optical pulsewidth T_c and τ is thus $T_c = \tau/L$. An example of such format is given in Fig. 2 with $M = 4$, $L = 7$, and $w = 3$.

The bit error rate performance of the above system has been studied in [3]–[6] for the case of $\lambda = 1$. We have found in [5], [6] that while keeping the transmitted information per photon (ρ nats/photon) fixed and maintaining the probability of error below some prescribed threshold $0 < \epsilon < 1$, the number of simultaneous users can be increased as desired by increasing the possible number of pulse positions M. Of course the price to be paid is the increase in the bandwidth in order to have same throughput per unit time. Another limitation is the maximum available number of subscribers (or signature codes) dictated by the design criteria of OOCs. Indeed for $\lambda = 1$, the number of available code sequences cannot exceed $(L-1)/(w(w-1))$. Obviously this limit can be tolerated by increasing the code length L leading to a narrower pulsewidth or increasing λ with L fixed.

Other more frequently encountered optical CDMA models [7]–[13] employ ON–OFF keying (OOK) instead of PPM before multiplexing. In this case the number of users cannot be increased *freely* without destroying the error rate threshold for any given value of ρ. The superiority of PPM-CDMA over OOK-CDMA in that sense makes it an attractive candidate in local area networks (LANs). Moreover, the traditional advantages in using PPM rather than OOK add to its preeminence, namely, it does not require a threshold in the detection process and is more efficient in utilizing the laser energy.

There have been some variants to the traditional optical CDMA systems in order to enhance its performance. Ohtsuki *et al.* [10] have proposed a synchronous optical CDMA system with double optical hard-limiters placed before and after the optical correlator. It has been shown that this system introduces an improvement in the performance over the system without optical hard-limiters as long as the number of users is not so large. In the case of asynchronous optical CDMA, Ohtsuki [11], [12] has shown that this improvement continues for all possible number of users. Another optical CDMA technique was introduced by Lam and Hussain [14] and generalized by Kwon [15]. In [15], multibits of the user data are mapped into shifted versions of the signature code allowing $\log L$ nats to be transmitted per sequence period. It has been shown that this system outperforms OOK scheme at the expense of increasing the hardware complexity.

In our study of optical PPM-CDMA networks we assume equi-probable data symbols, i.e., $\Pr\{D = d\} = 1/M$, $d \in \{0, 1, \ldots, M-1\}$. The transmitted information in nats per channel use is thus equal to $\log M$. For the sake of convenience, we will denote the last piece of information by m:

$$m \stackrel{\text{def}}{=} \log M \qquad \text{nats/channel use.}$$

Since both the maximum number of simultaneous users and the probability of error depend on M (or equivalently m), they may be denoted by N_m and $P_m[E]$, respectively.

Manuscript received July 26, 1999; revised May 23, 2000.

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Publisher Item Identifier S 0733-8724(00)08069-5.

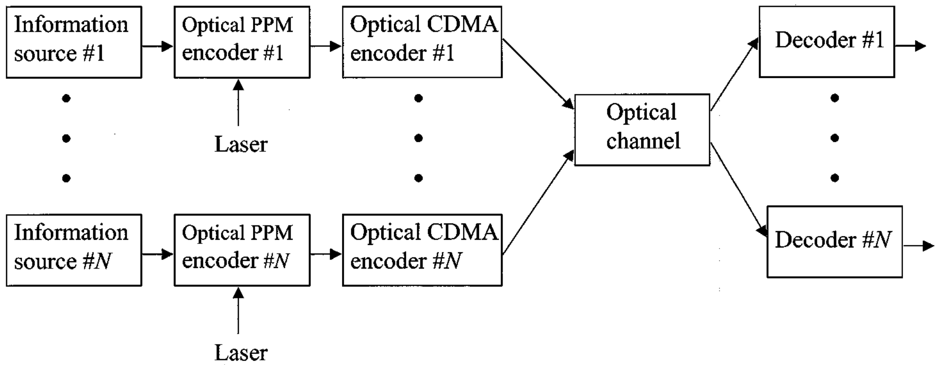

Fig. 1. An optical PPM-CDMA LAN.

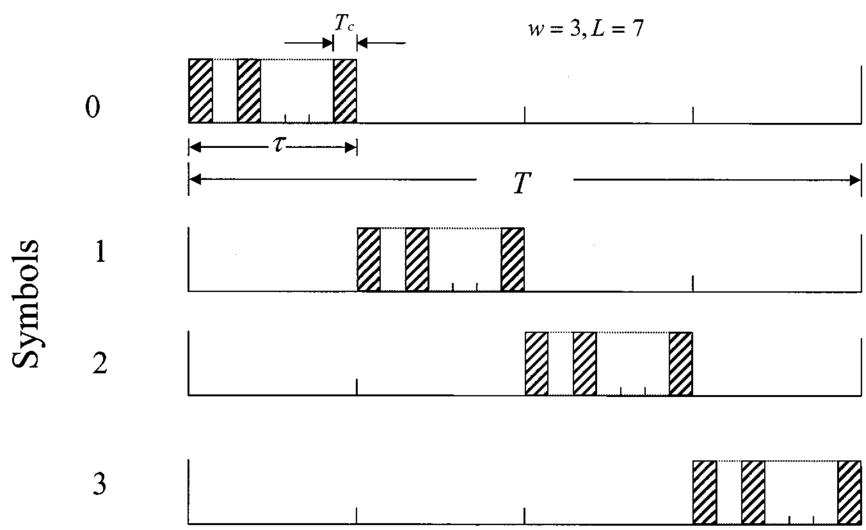

Fig. 2. An example of the transmitted signal formats of a single user in a PPM-CDMA system with $M = 4$, $L = 7$, and $w = 3$.

Our aim in this paper is to determine the *best* number of users that can be accommodated simultaneously by the optical PPM-CDMA system (for a given $\rho > 0$) while maintaining $P_m[E] < \epsilon$, any $0 < \epsilon < 1$. In our analysis, we let λ be any value in $\{1, 2 \ldots, w - 1\}$ allowing a larger subscribers limit since, in this case, the number of possible codes is upper bounded by [1]

$$\frac{(L-1)(L-2)\ldots(L-\lambda)}{w(w-1)\ldots(w-\lambda)}. \tag{1}$$

At this point, we would like to differentiate between the number of subscribers and the number of simultaneous users (or simply the number of users). The first refers to the number of available signature codes given by (1). The latter, however, refers to the possible number of active users that can communicate simultaneously and reliably. Of course the number of simultaneous users is in general less than the number of subscribers. The difference between these two numbers gives the number of idle users.

The asymptotic dependance of N_m on the pulse-position multiplicity M is not linear and can be assumed to have the following formula:

$$\frac{N_m}{L} = M^\theta \tag{2}$$

where θ is in general a function of L, w, λ, ρ, and ϵ. There are two important questions to be answered.

1) How does θ depend on the above parameters?
2) Does θ decrease to zero as $M \to \infty$?

The answer to the first question is important because it gives us an estimate to the number of simultaneous users that can communicate reliably using this network. The answer to the second question tells us whether all the subscribers can be loaded simultaneously to the channel and still achieve reliable communication or not. Indeed if θ is positive then by increasing M the number of users N_m can reach the number of subscribers given in (1).

In this paper we are able to find a lower bound to the exponent θ. This lower bound, in turn, identifies an achievable number of simultaneous users that can communicate reliably.

Further, we show that this exponent is positive as long as the transmitted information per photon ρ is greater than a certain amount. Our approach to characterize this exponent uses a rather complex mathematics as provided in subsequent sections. Another simple way to tackle this problem is to find the error rate in terms of the system parameters and show how these parameters tradeoff while keeping the error probability bounded. Although this simple approach shows how the number of users N_m changes with M, it cannot tell us the rate of change of N_m with respect to M, that is it does not answer exactly the above questions.

In our analysis, we take into account the effect of both the Poisson shot noise of the photodetector and the multiple-users interference. Whereas, the effect of both the dark current and thermal noise is neglected since their influence on the performance is minor. In fact the main source of limitation here is due to the multiple-users interference (especially for large number of users), and for sufficiently large laser energy the CDMA system reduces to a shot-noise limited one. Further, we derive our results under the assumption of chip-synchronous uniformly distributed relative delays among the receivers. This reduces the complexity of the analysis and gives more insights into the problem under consideration.

The formulation of the problem is given in Section II together with pertinent notation. Section III is devoted for some propositions and lemmas, the proofs of which are given in an Appendix. The main result along with some discussion appear in Section IV. The proof of our result is presented in Section V. Finally, the conclusion is given in Section VI.

II. Problem Statement and Preliminaries

A. Definitions

As M increases (with L fixed), the maximum number of simultaneous users N_m can be increased as well. Since we are interested in characterizing the rate of increase of N_m with respect to M, we define (in this subsection) some measures to this rate. Given an OOC with length L, weight w, and auto- and cross-correlation constraint $\lambda < w$, we have the following definitions.

1) The Users Rate: We define the rate of the maximum number of simultaneous users (or simply the users rate) as $(1/m)\log(N_m/L)$.

2) An ϵ-Achievable Users Rate: Given $0 < \epsilon < 1$, a nonnegative number R is said to be ϵ-achievable users rate for the PPM-CDMA optical channel if for every $\delta > 0$ and every sufficiently large m, we have

$$\frac{1}{m}\log\frac{N_m}{L} > R - \delta \qquad \text{with } P_m[E] \le \epsilon.$$

3) The ϵ-Users Strength: The supremum of ϵ-achievable users rates is called the ϵ-users strength $\theta(L, w, \lambda, \rho, \epsilon)$.

In the subsequent sections we intend to develop a lower bound to the ϵ-users strength defined above. As was shown in (2), this lower bound provides a measure to the maximum number of users that can communicate together with error rate not exceeding ϵ. Furthermore, we are able to show that $\theta(L, w, \lambda, \rho, \epsilon) > 0$ which signifies that it is possible to accommodate simultaneous users at a fixed positive rate while keeping the probability of error as small as desired. In other words, for finite values of L all the subscribers can be loaded simultaneously into the optical PPM-CDMA channel with arbitrary small error rate.

B. The Decision Rule

Each user counts the photons collected in the permissible chips (determined by the signature code) of every slot within the time frame. The number of the slot having the largest count is declared to be the transmitted symbol. We denote the photon count collected in slot $i \in \{0, 1, \ldots, M-1\}$ by Y_i. Symbol i is thus declared to be the true one if $Y_i > Y_j$ for every $j \ne i$. Hence, the probability of error can be written as

$$P_m[E] = \sum_{i=0}^{M-1} P_m[E|i]\Pr\{D=i\} = \frac{1}{M}\sum_{i=0}^{M-1} P_m[E|i]$$

where

$$P_m[E|i] = \Pr\{Y_j \ge Y_i, \quad \text{some } j \ne i | D = i\}.$$

It is obvious, because of the symmetry of the channel, that the last probability is independent of i. Consequently

$$P_m[E] = \Pr\{Y_j \ge Y_0, \quad \text{some } j \ne 0 | D = 0\}. \tag{3}$$

III. The Probability of Interference

Denote by κ_{it}, $i \in \{0, 1, \ldots, M-1\}$, $t \in \{1, 2, \ldots, \lambda\}$, the number of other users that cause interference at t pulse positions in slot i of the desired user. The following proposition and lemma, whose proofs appear in the Appendix, describe the joint probability distribution of the random vector $(\kappa_{i1}, \kappa_{i2}, \ldots, \kappa_{i\lambda})$.

Proposition 1: In a chip-synchronous optical PPM-CDMA channel employing OOCs with weight $w > 1$, length $L \ge w^2$, and auto- and cross-correlation constraint $\lambda < w$, if P_t, $t \in \{1, 2, \ldots, \lambda\}$, denotes the probability that a single user interferes with the desired user at t pulse positions then

$$\sum_{t=1}^{\lambda} t \cdot P_t = \frac{w^2}{ML} \tag{4}$$

where M denotes the pulse position multiplicity.

Lemma 1: For any $i \in \{0, 1, \ldots, M-1\}$, the random variables $\kappa_{i1}, \kappa_{i2}, \ldots, \kappa_{i\lambda}$ admit a multinomial joint distribution with parameters $N-1, P_1, P_2, \ldots, P_\lambda$:

$$\begin{aligned}\Pr\{\kappa_{i1} = l_{i1}, \ldots, \kappa_{i\lambda} = l_{i\lambda}\} &= \frac{(N-1)!}{l_{i1}!l_{i2}!\ldots l_{i\lambda}!(N-1-\sum_{t=1}^{\lambda} l_{it})!} \\ &\quad \times P_1^{l_{i1}}\ldots P_\lambda^{l_{i\lambda}}\left(1-\sum_{t=1}^{\lambda}P_t\right)^{N-1-\sum_{t=1}^{\lambda} l_{it}}\end{aligned}$$

where $(l_{i1}, l_{i2}, \ldots, l_{i\lambda})$ is a realization vector for $(\kappa_{i1}, \kappa_{i2}, \ldots, \kappa_{i\lambda})$.

IV. THE MAIN RESULT

A. Statement of the Result

We demonstrate in Theorem 1 below the main result on the characterization of the users strength for the aforementioned system. The proof of this theorem is given in Section V.

Theorem 1: In a chip-synchronous optical PPM-CDMA channel employing OOCs with weight $w > 1$, length $L \geq w^2$, and auto- and cross-correlation constraint $\lambda < w$, if the photodetector statistics are Poisson then the ϵ-users strength is lower bounded by the equation shown at the bottom of the page where $\lceil x \rceil$ denotes the smallest integer not less than x, ρ denotes the average information in nats/photon, and ρ_0 is the solution of

$$1 - \frac{\lambda}{w} \cdot \frac{e^{\rho_0} - 1}{\rho_0} = 0.$$

B. Discussion of the Result

The plot of the users strength versus ρ is given in Fig. 3 for certain system parameters. It is obvious that the users strength increases as ρ decreases. This is an expected result which demonstrates that for fixed system parameters (L, w, λ, ϵ, and M), increasing the average energy per optical pulse will allow more users to communicate reliably. Surprisingly, however, there is a threshold on ρ after which the users strength becomes independent of ρ and does not increase above a certain limit. That is, we cannot increase the number of simultaneous users (even if we increased the average energy) without disturbing the error constraint ϵ. Both the limit on the number of users and the threshold on ρ have been identified in Theorem 1. Obviously, if the number of users reached this limit and the average energy is increased, what we could gain is a decrease in the average error rate.

From the definition of the users strength we can estimate the maximum number of users that can communicate simultaneously and reliably as

$$\frac{N_m}{L} = e^{(\theta(L, w, \lambda, \rho, \epsilon) - o(m))m} = M^{\theta(L, w, \lambda, \rho, \epsilon) - o(m)} \tag{5}$$

where $o(m) \to 0$ as $m \to \infty$. Since $\theta(L, w, \lambda, \rho, \epsilon) > 0$ our system can accommodate any number of users by increasing the pulse position multiplicity M and keeping the average energy per pulse fixed. However for a given value of L, we cannot increase N_m as we wish because it is limited by the design criteria of OOCs as described in (1). Instead, our system can be loaded by all subscribers simultaneously and reliably. The best code (which offers the largest number of simultaneous users given L and w) is thus the one which maximizes (1). That is for finite L, the best code is that with $\lambda = w - 1$.

In the special case of ideal photodetector, where the Poisson shot noise can be neglected, the restriction on ρ in Theorem 1 can be removed. In fact the ideal system can be considered as the limit of the previous Poisson system with $\rho \to 0$. Instead, for a given $\rho > 0$ and a given error probability constraint, it provides an upper bound to the number of users in the Poisson system. We thus have the following corollary for the ideal system.

Corollary 1: In a chip-synchronous optical PPM-CDMA channel employing OOCs with weight $w > 1$, length $L \geq w^2$, and auto- and cross-correlation constraint $\lambda < w$, if the photodetector is ideal then the ϵ-users strength is lower bounded by

$$\theta(L, w, \lambda, \epsilon) \geq 1 - \left\lceil \frac{w}{\lambda} \right\rceil^{-1}$$

where $\epsilon \in (0, 1)$.

V. PROOF OF THEOREM 1

To simplify the notation, we omit the subscript m in N_m whenever there is no confusion. Let $\theta(L, w, \lambda, \rho, \epsilon) \geq 1 - (1/\alpha)$, where α is defined as shown in (6) at the bottom of the next page. Thus, for any $\delta > 0$ (small enough), N/L can be bounded as

$$\frac{N}{L} \leq M^{1-(1/\alpha)-\delta}. \tag{7}$$

Thus, it is enough to show that the corresponding error probability $P_m[E] \to 0$ as $M \to \infty$. It is obvious from (6) that if

$$\frac{\lambda \left\lceil \frac{w}{\lambda} \right\rceil}{\lambda \left\lceil \frac{w}{\lambda} \right\rceil - w + 1} \left(\log \frac{w}{w-1} - \frac{1}{w} \right) \leq \rho \leq w \left(\log \frac{w}{w-1} - \frac{1}{w} \right)$$

$$\theta(L, w, \lambda, \rho, \epsilon) \geq \begin{cases} 1 - \left\lceil \frac{w}{\lambda} \right\rceil^{-1}; & \text{if } \rho \leq \dfrac{\lambda \left\lceil \frac{w}{\lambda} \right\rceil}{\lambda \left\lceil \frac{w}{\lambda} \right\rceil - w + 1} \left(\log \dfrac{w}{w-1} - \dfrac{1}{w} \right) \\ 1 - \dfrac{\lambda}{w-1} \left[1 - \dfrac{\log \frac{w}{w-1} - \frac{1}{w}}{\rho} \right]; & \text{if } \dfrac{\lambda \left\lceil \frac{w}{\lambda} \right\rceil}{\lambda \left\lceil \frac{w}{\lambda} \right\rceil - w + 1} \left(\log \dfrac{w}{w-1} - \dfrac{1}{w} \right) \leq \rho \leq w \left(\log \dfrac{w}{w-1} - \dfrac{1}{w} \right) \\ 1 - \dfrac{\lambda}{w} \cdot \dfrac{e^{\rho} - 1}{\rho}; & \text{if } w \left(\log \dfrac{w}{w-1} - \dfrac{1}{w} \right) < \rho \leq \rho_0 \\ 0; & \text{if } \rho_0 \leq \rho \end{cases}$$

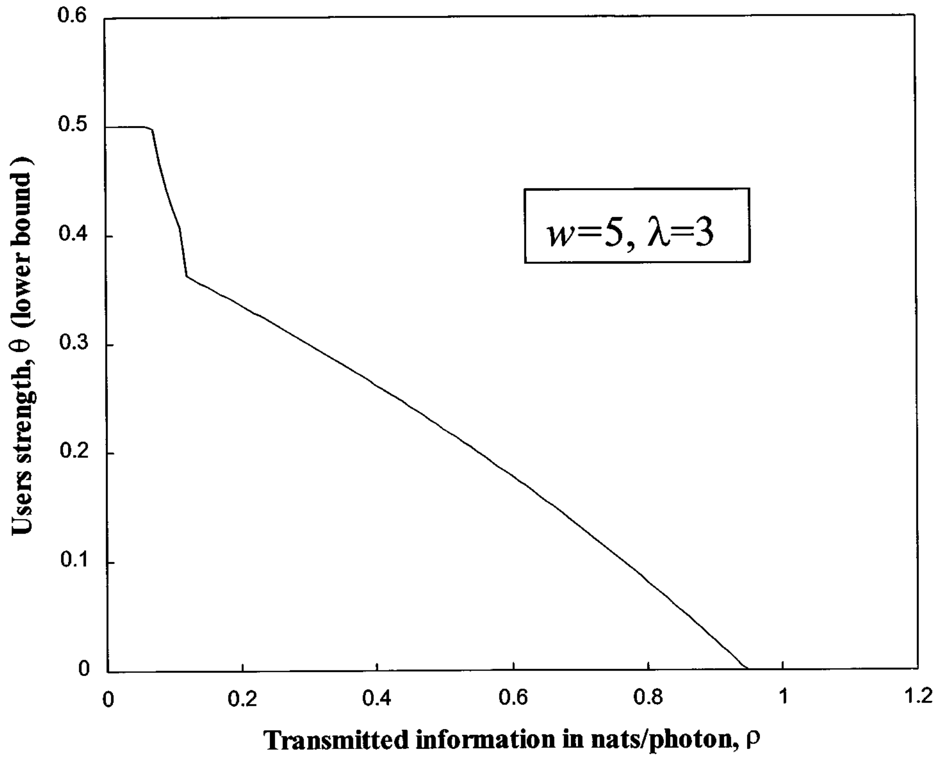

Fig. 3. A lower bound on the ϵ-users strength versus the transmitted information in nats per photon for the optical PPM-CDMA system with $w = 5$ and $\lambda = 3$.

then

$$\frac{w}{\lambda} = \frac{w-1}{\lambda}\left[1-\frac{1}{w}\right]^{-1} \leq \alpha \leq \frac{w-1}{\lambda}\left[1-\frac{\lambda\left\lceil\frac{w}{\lambda}\right\rceil - w+1}{\lambda\left\lceil\frac{w}{\lambda}\right\rceil}\right]^{-1} = \left\lceil\frac{w}{\lambda}\right\rceil \tag{8}$$

and if $w(\log(w/(w-1)) - (1/w)) < \rho \leq \rho_0$, then

$$1 \leq \alpha = \frac{w}{\lambda}\cdot\frac{\rho}{e^{\rho}-1} \leq \frac{w}{\lambda}. \tag{9}$$

The first inequality results from the definition of ρ_0 whereas the last inequality results from the fact that $e^{\rho} \geq 1+\rho$. We denote by I_i, $i \in \{0, 1, \ldots, M-1\}$, the number of interfering pulses in slot i:

$$I_i \overset{\text{def}}{=} \sum_{t=1}^{\lambda} t\cdot\kappa_{it}. \tag{10}$$

The error probability given in (3) can be written as

$$\begin{aligned} P_m[E] &= \Pr\{Y_j \geq Y_0, \quad \text{some } j \neq 0 | D=0\} \\ &= \Pr\{Y_j \geq Y_0, \quad \text{some } j \neq 0,\, I_i \geq w \\ &\qquad \text{some } i \neq 0 | D = 0\} \\ &\quad + \Pr\{Y_j \geq Y_0, \quad \text{some } j \neq 0,\, I_0 \neq 0 \\ &\qquad I_i < w \quad \forall i \neq 0 | D=0\} \\ &\quad + \Pr\{Y_j \geq Y_0, \quad \text{some } j \neq 0,\, I_0 = 0 \\ &\qquad I_i < w \quad \forall i \neq 0 | D=0\} \end{aligned}$$

$$\alpha \overset{\text{def}}{=} \begin{cases} \left\lceil\frac{w}{\lambda}\right\rceil; & \text{if } 0 < \rho \leq \dfrac{\lambda\left\lceil\frac{w}{\lambda}\right\rceil}{\lambda\left\lceil\frac{w}{\lambda}\right\rceil - w+1}\left(\log\dfrac{w}{w-1} - \dfrac{1}{w}\right) \\ \dfrac{w-1}{\lambda}\left[1-\dfrac{\log\frac{w}{w-1}-\frac{1}{w}}{\rho}\right]^{-1}; & \text{if } \dfrac{\lambda\left\lceil\frac{w}{\lambda}\right\rceil}{\lambda\left\lceil\frac{w}{\lambda}\right\rceil - w+1}\left(\log\dfrac{w}{w-1} - \dfrac{1}{w}\right) \leq \rho \leq w\left(\log\dfrac{w}{w-1}-\dfrac{1}{w}\right) \\ \dfrac{w}{\lambda}\cdot\dfrac{\rho}{e^{\rho}-1}; & \text{if } w\left(\log\dfrac{w}{w-1}-\dfrac{1}{w}\right) < \rho \leq \rho_0. \end{cases} \tag{6}$$

$$\begin{aligned}&\le \Pr\{I_i \ge w, \quad \text{some } i \ne 0\} + \Pr\{I_0 \ne 0\}\\ &\quad + \Pr\{Y_j \ge Y_0, \quad \text{some } j \ne 0,\, I_0 = 0\\ &\qquad I_i < w \quad \forall i \ne 0 | D = 0\}\\ &= P_m[E1] + P_m[E2] + P_m[E3]\end{aligned}$$

where

$$\begin{aligned}P_m[E1] &\stackrel{\text{def}}{=} \Pr\{I_i \ge w, \quad \text{some } i \ne 0\}\\ P_m[E2] &\stackrel{\text{def}}{=} \Pr\{I_0 \ne 0\}\\ P_m[E3] &\stackrel{\text{def}}{=} \Pr\{Y_j \ge Y_0, \quad \text{some } j \ne 0\\ &\qquad I_0 = 0,\, I_i < w \quad \forall i \ne 0 | D = 0\}.\end{aligned}$$

Here, the second error event $E2$ is due to neglecting the interference in the data slot, whereas the first error event $E1$ is due to the multiple-users interference in other slots, and the last error event $E3$ is due to the shot noise process of the photodetector. Of course for the case of ideal photodetectors (cf., Corollary 1), only the first error event $E1$ contributes. We now show that $P_m[Ei] \to 0$ as $M \to \infty$ for every $i \in \{1, 2, 3\}$.

$$\begin{aligned}P_m[E2] &= \Pr\{I_0 \ne 0\} = \Pr\{I_0 \ge 1\} \le E\{I_0\}\\ &= E\left\{\sum_{t=1}^{\lambda} t \cdot \kappa_{0t}\right\} = \sum_{t=1}^{\lambda} t(N-1)P_t = (N-1)\frac{w^2}{ML}\\ &\le M^{1-(1/\alpha)-\delta}\frac{w^2}{M} = w^2 M^{-(1/\alpha)-\delta}. \qquad (11)\end{aligned}$$

Here, $E\{\cdot\}$ denotes the expected value, the first inequality is justified by the use of the Markov inequality, the expectation evaluation is immediate from Lemma 1, and the last inequality is because of (7). Consequently $P_m[E2] \to 0$ as $M \to \infty$.

$$\begin{aligned}P_m[E1] &= \Pr\{I_i \ge w, \quad \text{some } i \ne 0\}\\ &\le (M-1)\Pr\{I_1 \ge w\}\\ &= (M-1)\Pr\left\{\sum_{t=1}^{\lambda} t \cdot \kappa_{1t} \ge w\right\}\end{aligned}$$

where a union bound has been used in the last inequality and I_1 has been substituted from (10) in the last equality. Since $t \le \lambda$, we can further write

$$\begin{aligned}P_m[E1] &\le (M-1)\Pr\left\{\lambda\sum_{t=1}^{\lambda} \kappa_{1t} \ge w\right\}\\ &= (M-1)\Pr\left\{\sum_{t=1}^{\lambda} \kappa_{1t} \ge \left\lceil\frac{w}{\lambda}\right\rceil\right\}\\ &= (M-1)\Pr\left\{z^{\kappa_{11}+\cdots+\kappa_{1\lambda}} \ge z^{\nu}\right\}\end{aligned}$$

for any $z \ge 1$ and $\nu \stackrel{\text{def}}{=} \lceil w/\lambda \rceil$. Using the Markov inequality again and applying Lemma 1, we get

$$\begin{aligned}P_m[E1] &\le (M-1)z^{-\nu}E\left\{z^{\kappa_{11}+\cdots+\kappa_{1\lambda}}\right\}\\ &= (M-1)z^{-\nu}\left(1 - \sum_{t=1}^{\lambda} P_t + \sum_{t=1}^{\lambda} zP_t\right)^{N-1}.\end{aligned}$$

Taking the logarithm of the above error rate, yields

$$\begin{aligned}\log P_m[E1] &\le \log(M-1) - \nu\log z + (N-1)\\ &\quad \times \log\left(1 - \sum_{t=1}^{\lambda} P_t + z\sum_{t=1}^{\lambda} P_t\right)\\ &\le \log M - \nu\log z + Nz\sum_{t=1}^{\lambda} P_t.\end{aligned}$$

Indeed, since $\log(1 - x + y) \le y$ for any real numbers y and $x \ge 0$. Noticing from Proposition 1 that

$$\sum_{t=1}^{\lambda} P_t \le \sum_{t=1}^{\lambda} t \cdot P_t = \frac{w^2}{ML}$$

would yield

$$\log P_m[E1] \le \log M - \nu\log z + Nz\frac{w^2}{ML}.$$

Thus

$$\log P_m[E1] \le \log M + \min_{z \ge 1}\left\{Nz\frac{w^2}{ML} - \nu\log z\right\}.$$

It is easy to check that the last minimum occurs at $z = (\nu ML/w^2 N)$. Consequently

$$\begin{aligned}P_m[E1] &\le M\left(\frac{\nu ML}{w^2 N}\right)^{-\nu} e^{\nu} = \left(\frac{w^2 e}{\nu}\right)^{\nu}\left(\frac{N}{M^{1-\frac{1}{\nu}}L}\right)^{\nu}\\ &\le \left(\frac{w^2 e}{\nu}\right)^{\nu}\left(\frac{M^{1-\frac{1}{\alpha}-\delta}}{M^{1-\frac{1}{\nu}}}\right)^{\nu}\\ &= \left(\frac{w^2 e}{\nu}\right)^{\nu}\left(M^{\frac{1}{\nu}-\frac{1}{\alpha}-\delta}\right)^{\nu} \le \left(\frac{w^2 e}{\nu}\right)^{\nu} M^{-\delta\nu}.\end{aligned} \qquad (12)$$

The first inequality holds because of (7) and the last inequality holds because $\alpha \le \lceil w/\lambda \rceil = \nu$, cf., (6), (8), and (9). Hence, $P_m[E1] \to 0$ as $M \to \infty$ as well. Now it remains to show that $P_m[E3] \to 0$ as $M \to \infty$ to complete the proof.

$$\begin{aligned}P_m[E3] &= \Pr\{Y_j \ge Y_0, \quad \text{some } j \ne 0\\ &\qquad I_0 = 0,\, I_i < w \quad \forall i \ne 0 | D = 0\}\\ &= \sum_{y_0} \Pr\{I_0 = 0 | D = 0\}\\ &\quad \times \Pr\{Y_0 = y_0 | D = 0,\, I_0 = 0\}\\ &\quad \times \Pr\{Y_j \ge y_0, \quad \text{some } j \ne 0\\ &\qquad I_i < w \quad \forall i \ne 0 | D = 0,\, I_0 = 0,\, Y_0 = y_0\}.\end{aligned}$$

Since both Y_j and I_i are independent of Y_0 given I_0 and D, we get, for any $0 < s \le 1$

$$\begin{aligned}P_m[E3] &= \sum_{y_0} \Pr\{Y_0 = y_0 | D = 0,\, I_0 = 0\}\\ &\quad \times \Pr\{Y_j \ge y_0, \quad \text{some } j \ne 0\\ &\qquad I_0 = 0,\, I_i < w \quad \forall i \ne 0 | D = 0\}\\ &\le \sum_{y_0} \Pr\{Y_0 = y_0 | D = 0,\, I_0 = 0\}\\ &\quad \times \Pr\{Y_j \ge y_0, \quad \text{some } j \ne 0\\ &\qquad I_i < w \quad \forall i \ne 0 | D = 0\}\end{aligned}$$

$$\begin{aligned}
&\leq \sum_{y_0} \Pr\{Y_0 = y_0 | D = 0,\, I_0 = 0\} \\
&\qquad \times [\Pr\{Y_j \geq y_0, \quad \text{some } j \neq 0, \\
&\qquad\qquad I_i < w \quad \forall\, i \neq 0 | D = 0\}]^s \\
&\leq (M-1)^s \sum_{y_0} \Pr\{Y_0 = y_0 | D = 0,\, I_0 = 0\} \\
&\qquad \times [\Pr\{Y_1 \geq y_0,\, I_i < w \quad \forall\, i \neq 0 | D = 0\}]^s .
\end{aligned}$$

The last inequality is justified by using a simple union bound. Since the set $[I_i < w \quad \forall\, i \neq 0]$ is a subset of $[I_1 < w]$, $P_m[E3]$ can further be increased as

$$\begin{aligned}
P_m[E3] &\leq (M-1)^s \sum_{y_0} \Pr\{Y_0 = y_0 | D = 0,\, I_0 = 0\} \\
&\qquad \times [\Pr\{Y_1 \geq y_0,\, I_1 < w | D = 0\}]^s \\
&= (M-1)^s \sum_{y_0} \Pr\{Y_0 = y_0 | D = 0,\, I_0 = 0\} P_m^s[E4]
\end{aligned} \tag{13}$$

where

$$P_m[E4] \overset{\text{def}}{=} \Pr\{Y_1 \geq y_0,\, I_1 < w | D = 0\}.$$

By substituting I_1 from (10) and using Chernoff inequality, we can estimate $P_m[E4]$ as follows. For any $z \geq 1$

$$\begin{aligned}
P_m[E4] &= \sum_{\substack{l_{11},\, \ldots,\, l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} \Pr\{\kappa_{11} = l_{11},\, \ldots,\, \kappa_{1\lambda} = l_{1\lambda}\} \\
&\qquad \times \Pr\{Y_1 \geq y_0 | D = 0,\, \kappa_{11} = l_{11},\, \ldots,\, \kappa_{1\lambda} = l_{1\lambda}\} \\
&\leq z^{-y_0} \sum_{\substack{l_{11},\, \ldots,\, l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} \Pr\{\kappa_{11} = l_{11},\, \ldots,\, \kappa_{1\lambda} = l_{1\lambda}\} \\
&\qquad \times E\{z^{Y_1} | D = 0,\, \kappa_{11} = l_{11},\, \ldots,\, \kappa_{1\lambda} = l_{1\lambda}\}.
\end{aligned}$$

But Y_1 is a conditional Poisson random variable with mean

$$E\{Y_1 | D = 0,\, \kappa_{11} = l_{11},\, \ldots,\, \kappa_{1\lambda} = l_{1\lambda}\} = \mu \sum_{t=1}^{\lambda} t l_{1t},$$

where μ is the average photon count per pulse, given by $\mu = \log M / \rho w$. Thus

$$\begin{aligned}
P_m[E4] &= z^{-y_0} \sum_{\substack{l_{11},\ldots,l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} \Pr\{\kappa_{11} = l_{11},\, \ldots,\, \kappa_{1\lambda} = l_{1\lambda}\} \\
&\qquad \times \exp\left[\mu(z-1) \sum_{t=1}^{\lambda} t l_{1t}\right]
\end{aligned}$$

Invoking Lemma 1 and substituting for μ yield

$$\begin{aligned}
P_m[E4] &\leq z^{-y_0} \sum_{\substack{l_{11},\, \ldots,\, l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} \frac{(N-1)!}{l_{11}! \ldots l_{1\lambda}! (N-1-\sum_{t=1}^{\lambda} l_{1t})!} \\
&\qquad \times P_1^{l_{11}} \ldots P_\lambda^{l_{1\lambda}} \left(1 - \sum_{t=1}^{\lambda} P_t\right)^{N-1-\sum_{t=1}^{\lambda} l_{1t}} \\
&\qquad \times M^{((z-1)/\rho w) \sum_{t=1}^{\lambda} t l_{1t}}.
\end{aligned}$$

Since $P_t \leq (w^2/tML) \leq (w^2/ML)$ (cf., Proposition 1), $1 - \sum_{t=1}^{\lambda} P_t \leq 1$, $l_{1t}! \geq 1$, and

$$(N-1)!/(N-1-\sum_{t=1}^{\lambda} l_{1t})! \leq (N-1)^{\sum_{t=1}^{\lambda} l_{1t}}$$

we obtain

$$\begin{aligned}
P_m[E4] &\leq z^{-y_0} \sum_{\substack{l_{11},\, \ldots,\, l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} (N-1)^{\sum_{t=1}^{\lambda} l_{1t}} \\
&\qquad \times \left(\frac{w^2}{ML}\right)^{\sum_{t=1}^{\lambda} l_{1t}} M^{((z-1)/\rho w) \sum_{t=1}^{\lambda} t l_{1t}} \\
&\leq z^{-y_0} \cdot w^{2w} \sum_{\substack{l_{11},\, \ldots,\, l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} \\
&\qquad \times \left(\frac{N}{ML}\right)^{\sum_{t=1}^{\lambda} l_{1t}} M^{((z-1)/\rho w) \sum_{t=1}^{\lambda} t l_{1t}}
\end{aligned}$$

where the constraint $\sum_{t=1}^{\lambda} t l_{1t} < w$ ensures that $\sum_{t=1}^{\lambda} l_{1t} < w$. Using (7) we can write

$$\begin{aligned}
P_m[E4] &\leq z^{-y_0} \cdot w^{2w} \sum_{\substack{l_{11},\, \ldots,\, l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} \\
&\qquad \times M^{-((1/\alpha)+\delta) \sum_{t=1}^{\lambda} l_{1t}} M^{((z-1)/\rho w) \sum_{t=1}^{\lambda} t l_{1t}}.
\end{aligned}$$

But the number of terms in the main summation of the last inequality cannot exceed w^λ, thus

$$\begin{aligned}
P_m[E4] &\leq w^{2w} \cdot z^{-y_0} \cdot w^\lambda \max_{\substack{l_{11},\, \ldots,\, l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} \\
&\qquad \times M^{((z-1)/\rho w) \sum_{t=1}^{\lambda} t l_{1t} - ((1/\alpha)+\delta) \sum_{t=1}^{\lambda} l_{1t}} \\
&\leq w^{2w} \cdot z^{-y_0} \cdot w^\lambda \max_{\substack{l_{11},\, \ldots,\, l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} \\
&\qquad \times M^{((z-1)/\rho w) \sum_{t=1}^{\lambda} t l_{1t} - ((1/\alpha)+\delta) \sum_{t=1}^{\lambda} (t/\lambda) l_{1t}} \\
&= w^{2w+\lambda} \cdot z^{-y_0} \max_{\substack{l_{11},\ldots,l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} < w}} \\
&\qquad \times M^{((z-1)/\rho w) - (1/\alpha\lambda) - (\delta/\lambda)) \sum_{t=1}^{\lambda} t l_{1t}}.
\end{aligned} \tag{14}$$

The last inequality holds since $t \leq \lambda$. Consider the following maximization problem on the exponent of M:

$$\mathcal{X} \overset{\text{def}}{=} \max_{\substack{l_{11},\ldots,l_{1\lambda}: \\ \sum_{t=1}^{\lambda} t l_{1t} \leq w-1}} \left(\frac{z-1}{\rho w} - \frac{1}{\alpha\lambda} - \frac{\delta}{\lambda}\right) \sum_{t=1}^{\lambda} t l_{1t}.$$

If the bracketed term is positive, then the maximum is achieved at the boundary, i.e., when $\sum_{t=1}^{\lambda} t l_{1t} = w - 1$, otherwise it is achieved when $l_{1t} = 0$, $t \in \{1, 2, \ldots, \lambda\}$. Thus, the solution of the maximization problem is as follows:

$$\mathcal{X} = \begin{cases} (w-1) \left(\dfrac{z-1}{\rho w} - \dfrac{1}{\alpha\lambda} - \dfrac{\delta}{\lambda}\right); & \text{if } \dfrac{z-1}{\rho w} \geq \dfrac{1}{\alpha\lambda} + \dfrac{\delta}{\lambda} \\ 0; & \text{if } 0 \leq \dfrac{z-1}{\rho w} \leq \dfrac{1}{\alpha\lambda} + \dfrac{\delta}{\lambda}. \end{cases} \tag{15}$$

Substituting in (14) and then in (13) would yield

$$P_m[E4] \le w^{2w+\lambda} \cdot z^{-y_0} M^{\mathcal{X}}$$

and

$$\begin{aligned} P_m[E3] &\le w^{s(2w+\lambda)} M^s M^{s\mathcal{X}} \sum_{y_0} z^{-sy_0} \\ &\quad \times \Pr\{Y_0 = y_0 | D = 0,\, I_0 = 0\} \\ &= w^{s(2w+\lambda)} M^s M^{s\mathcal{X}} E\left\{z^{-sY_0} | D = 0,\, I_0 = 0\right\}. \end{aligned}$$

But Y_0 is also a conditional Poisson random variable with mean

$$E\{Y_0 | D = 0,\, I_0 = 0\} = \mu w = \frac{\log M}{\rho}.$$

Hence

$$\begin{aligned} P_m[E3] &\le w^{s(2w+\lambda)} M^s M^{s\mathcal{X}} \exp[\mu w(z^{-s} - 1)] \\ &= w^{s(2w+\lambda)} M^s M^{s\mathcal{X}} M^{(z^{-s}-1)/\rho} \\ &= w^{s(2w+\lambda)} M^s M^{s\mathcal{X}} M^{-(s/\rho)\cdot((1-z^{-s})/s)} \\ &= w^{s(2w+\lambda)} M^s M^{s\mathcal{X}} M^{-(s/\rho)(\log z - o(s))} \end{aligned}$$

where $o(s) \searrow 0$ as $s \to 0$. Indeed the last equality can be justified by using Maclaurin series of $(1 - z^{-s})/s$. For sufficiently large values of M and small values of s, we can write

$$\begin{aligned} -\frac{\log P_m[E3]}{s\dfrac{\log M}{\rho}} &\ge \log z - \rho - \rho\mathcal{X} - o(s) - \frac{\rho(2w+\lambda)\log w}{\log M} \\ &\ge \log z - \rho - \rho\mathcal{X} - \xi \end{aligned} \tag{16}$$

for some $\xi > 0$ arbitrary small. Now consider two cases

Case A: This case occurs whenever

$$w\left(\log\frac{w}{w-1} - \frac{1}{w}\right) < \rho \le \rho_0.$$

It corresponds to the last case in (6), that is

$$\alpha = \frac{w}{\lambda} \cdot \frac{\rho}{e^\rho - 1}. \tag{17}$$

Choose $z \ge 1$ such that

$$\frac{z-1}{\rho w} \le \frac{1}{\alpha\lambda} + \frac{\delta}{\lambda}.$$

Thus from (15) and (16), we get $\mathcal{X} = 0$ and

$$-\frac{\log P_m[E3]}{s\frac{\log M}{\rho}} \ge \log z - \rho - \xi$$

respectively. The optimum value of z, for the last inequality to be tightest, is achieved at the boundary of the above specified range, i.e., $z = 1 + \rho w((1/\alpha\lambda) + (\delta/\lambda))$. Consequently

$$-\frac{\log P_m[E3]}{s\dfrac{\log M}{\rho}} \ge \log\left(1 + \frac{\rho w}{\alpha\lambda} + \frac{\delta\rho w}{\lambda}\right) - \rho - \xi.$$

Substituting for α from (17), we obtain

$$-\frac{\log P_m[E3]}{s\dfrac{\log M}{\rho}} \ge \log\left(e^\rho + \frac{\delta\rho w}{\lambda}\right) - \rho - \xi > 0.$$

Since the right hand side is positive for small values of ξ, $P_m[E3] \to 0$ as $M \to \infty$.

Case B: This case occurs whenever

$$0 \le \rho \le w\left(\log\frac{w}{w-1} - \frac{1}{w}\right).$$

It corresponds to the first two cases in (6), that is

$$\frac{w}{\lambda} \le \alpha \le \lceil \frac{w}{\lambda} \rceil. \tag{18}$$

Choose $z \ge 1$ such that

$$\frac{z-1}{\rho w} \ge \frac{1}{\alpha\lambda} + \frac{\delta}{\lambda}.$$

Thus, from (15) and (16), we have

$$\mathcal{X} = (w-1)\left(\frac{z-1}{\rho w} - \frac{1}{\alpha\lambda} - \frac{\delta}{\lambda}\right)$$

and

$$\begin{aligned} -\frac{\log P_m[E3]}{s\dfrac{\log M}{\rho}} \ge &\max_{\substack{z\ge 1:\\ ((z-1)/\rho w) \ge (1/\alpha\lambda) + (\delta/\lambda)}} \\ &\times \left\{\log z - \rho - \rho(w-1)\left(\frac{z-1}{\rho w} - \frac{1}{\alpha\lambda} - \frac{\delta}{\lambda}\right) - \xi\right\} \end{aligned}$$

respectively. The right-hand side (RHS) can be decreased if we choose $z = (w/(w-1)) + (\delta\rho w/\lambda)$. Indeed, this value satisfies the last maximization constraint since

$$\begin{aligned} \frac{z-1}{\rho w} &= \frac{1}{\rho w(w-1)} + \frac{\delta}{\lambda} \\ &\ge \frac{1}{w^2(w-1)\left(\log\dfrac{w}{w-1} - \dfrac{1}{w}\right)} + \frac{\delta}{\lambda} \\ &> \frac{1}{w} + \frac{\delta}{\lambda} \ge \frac{1}{\alpha\lambda} + \frac{\delta}{\lambda} \end{aligned}$$

where we have used the fact that $(\forall y < 1) \log y < y - 1$ to justify the last strict inequality. Consequently

$$\begin{aligned} -\frac{\log P_m[E3]}{s\dfrac{\log M}{\rho}} &\ge \log\left(\frac{w}{w-1} + \frac{\delta\rho w}{\lambda}\right) - \rho \\ &\quad - \rho(w-1)\left(\frac{1}{\rho w(w-1)} - \frac{1}{\alpha\lambda}\right) - \xi \\ &= \log\left(\frac{w}{w-1} + \frac{\delta\rho w}{\lambda}\right) - \frac{1}{w} \\ &\quad - \rho\left(1 - \frac{w-1}{\alpha\lambda}\right) - \xi. \end{aligned} \tag{19}$$

At this point, two more cases may arise. First, $\alpha = \lceil w/\lambda \rceil$, which corresponds to the first case in (6). Here, the RHS is positive as long as

$$0 < \rho \le \frac{\lambda\left\lceil\dfrac{w}{\lambda}\right\rceil}{\lambda\left\lceil\dfrac{w}{\lambda}\right\rceil - w + 1}\left(\log\frac{w}{w-1} - \frac{1}{w}\right).$$

Second

$$\alpha = \frac{w-1}{\lambda}\left[1 - \frac{\log\frac{w}{w-1} - \frac{1}{w}}{\rho}\right]^{-1}$$

which corresponds to the second case in (6). Here, the RHS equals $\log((w/(w-1))+(\delta\rho w/\lambda))-\log(w/(w-1))-\xi$, which is always positive for sufficiently small values of ξ. Of course the positivity of the right hand side ensures that $P_m[E3] \to 0$ as $M \to \infty$, which completes the proof of Theorem 1. □

VI. Extension and Concluding Remarks

In this paper the concept of users strength in direct-detection optical PPM-CDMA channels has been introduced. In fact this notion is a measure to the theoretical limit of the maximum number of users that can communicate simultaneously at an asymptotically ($M \to \infty$) zero error rate. One theorem, which characterizes this users strength, has been presented taking into account both the Poisson statistics of the photodetectors and the multiple-users interference. The effect of both the dark current and thermal noise has been neglected, since their influence is minor. It has been found that the users strength is independent of ρ whenever it is below some specified threshold. That is (for fixed $L, w, \lambda, \epsilon, M$) decreasing ρ below that threshold cannot increase the number of users and will lead only to a decrease in the average error rate. On the other hand, if ρ is above the threshold, one can increase the number of users and/or decrease the average error rate by decreasing ρ.

It has been shown in the discussion that, for a given value of L, all the subscribers can be loaded simultaneously into the optical PPM-CDMA channel with arbitrary small error rate. Further, it has been argued that the best code in this case is the one having $\lambda = w-1$. On the other hand if L is allowed to increase *freely* ($L \to \infty$) two cases may arise.

Case 1: If $\lambda = 1$, then (1) reduces to $(L-1)/w(w-1)$ and similar to the finite case all subscribers can be accommodated simultaneously.

Case 2: If $\lambda \geq 2$, however, one cannot accommodate all subscribers with asymptotically zero error rate. Indeed to have $P_m[E] \to 0$ one should restrict N_m as in (5) which is probably less than the bound in (1). Now the best code is the one which maximizes (5) subject to the constraint that $2 \leq \lambda < w$. Thus

$$\lambda = \arg\left\{\max_{\substack{\lambda:\\ 2\leq\lambda<w}} \theta(L, w, \lambda, \rho, \epsilon)\right\} = 2.$$

Our results on the characterization of the users strength given in both Theorem 1 and Corollary 1 are only partial, i.e., they only provide lower bounds. Recently [16], we have developed an upper bound to the users strength for the case of ideal photodiodes. It turned out that this upper bound is exactly the same as that given in Corollary 1 and in the first case of Theorem 1. Thus, with this recent result, the complete characterization is now settled for the ideal case.

Appendix
Proofs of Proposition 1 and Lemma 1

A. Proof of Proposition 1

Consider an interference random variable T. This random variable takes values in the set $\{0, 1, \dots, \lambda\}$ with probability distribution:

$$P_T(t) = \begin{cases} P_t; & \text{if } t \in \{1, 2, \dots, \lambda\} \\ 1 - \sum_{t=1}^{\lambda} P_t; & \text{if } t = 0. \end{cases}$$

Thus, the expected value of T is given by

$$E\{T\} = \sum_{t=1}^{\lambda} t \cdot P_t.$$

On the other hand

$$E\{T\} = \frac{1}{M} E\left\{\sum_{j=1}^{L} a_j^1 a_{j\ominus U}^2\right\}$$

where $\{a_j^i\}_{j=1}^L, i \in \{1, 2\}$ denote the code words of users 1 and 2, respectively, $\oplus$ denotes addition modulo L, and U denotes a shift random variable. Assuming chip-synchronous codes, U is a uniform random variable that takes values from the discrete set $\{0, 1, \dots, L-1\}$. Performing the last expectation yields $E\{T\} = w^2/ML$, which completes the proof of the proposition. □

B. Proof of Lemma 1

Fix $i \in \{0, 1, \dots, M-1\}$. Let A_t, $t \in \{1, 2, \dots, \lambda\}$, be the event that a single user interferes with the desired user at t pulse positions within slot i. Each of these disjoint events occurs with probability P_t. Further, let A_0 be the event that a single user does not interfere at all with slot i of the desired user. Obviously, A_0 occurs with probability $1-\sum_{t=1}^{\lambda} P_t$. Thus, if l_{it}, $t \in \{1, 2, \dots, \lambda\}$ corresponds to the number of times that A_t, $t \in \{1, 2, \dots, \lambda\}$, occurs, then A_0 occurs $N-1-\sum_{t=1}^{\lambda} l_{it}$ times. This formulation leads to the well-known multinomial distribution. □

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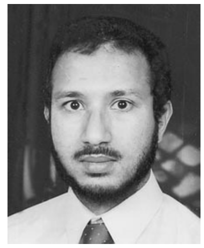

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