A Performance Analysis of Optical Overlapping PPM-CDMA Communication Systems

Hossam M. H. Shalaby, *Member, IEEE*

***Abstract*—Direct-detection optical code-division multiple-access (CDMA) systems employing overlapping pulse-position modulation (OPPM) schemes are proposed. Both upper and lower bounds on the bit error rate (BER) are derived taking into account the effect of both multiple-user interference and receiver shot noise. The photodiodes' dark currents are neglected since their effect is minor. The throughput limitation of this system is evaluated as well. Performance characteristics are then compared to optical CDMA systems employing traditional ON–OFF keying (OOK) and pulse-position modulation (PPM) schemes. It is shown that under fixed data rate and chip time, OPPM-CDMA system superperforms both traditional systems. Moreover, it is shown that the throughput limitation of OPPM-CDMA is almost 6.7 times greater than that of OOK-CDMA.**

***Index Terms*—Code division multiple access (CDMA), direct-detection optical channel, ON–OFF keying (OOK), optical CDMA, overlapping pulse-position modulation (OPPM), pulse-position modulation (PPM), spread spectrum.**

I. Introduction

In the few recent years, an increasing interest has been given to the design and analysis of optical communication networks. This results from the enormous development of data communication networks all over the world and the impressive amount of data to be transmitted over these networks. Indeed, in order to support image-based and multimedia applications, local- and wide-area networks (LAN's and WAN's) must be able to endow data at rates of hundreds of megabits/second to desktops. Current monotonous networks will not accommodate such a large amount of data in the future. However, contemporary technology in optical fiber networks makes it possible to endow immense amount of transmission capacity economically.

In optical data communication networks, multiple access may be implemented by many methods such as time-division multiple-access (TDMA), wavelength division multiple-access (WDMA), code-division multiple-access (CDMA), etc. CDMA has several advantages over other techniques. Namely, unnecessary time synchronization and frequency management, simple communication protocols, complete utilization of the entire time-frequency domain by each subscriber, flexibility in network design, and security against interception. One limitation, however, of binary CDMA is that it has less capacity than TDMA.

Optical CDMA systems with either binary ON–OFF keying (OOK) or M-ary pulse-position modulation (PPM) schemes have been appeared in literature [1]–[9]. It has been shown that under fixed throughput and chip time, there is no advantage in using PPM in place of OOK, but PPM becomes superior to OOK if the average power rather than the chip time is the constraining factor.

Recently, interest has been given to overlapping pulse-position modulation (OPPM) as an alternative signaling format to the conventional pulse-position modulation in direct-detection optical channels [10]–[15]. This type of signaling can be considered as a generalization to PPM signaling where overlapping is allowed between pulse positions. The reason to prefer OPPM over PPM is that the throughput (bits/s) can be improved without decreasing the pulsewidth. Moreover, OPPM retains the advantages of PPM in terms of implementation simplicity. Indeed the transmitter involves only time delaying of the optical pulse, and the receiver does not require knowledge of the signal or noise power.

In this paper, we suggest employing OPPM in optical CDMA channels. Our aim is to tolerate the throughput and the users capacity limitations of both OOK-CDMA and PPM-CDMA systems. We also compare the bit error rate performance of the suggested scheme with both the traditional schemes.

In our theoretical analysis we consider the effect of both multiple-user interference and receiver shot noise. In order to have some insight on the results obtained, we assume chip-synchronous uniformly distributed relative delays among the transmitters. In the numerical analysis, we employ optical orthogonal codes (OOC's) [1] as the signature code sequences. To have minimal interference between the users we adopt OOC's with periodic cross-correlations and out-of-phase periodic autocorrelations that are bounded only by one.

The remaining of our paper is organized as follows. Section II is devoted for the description of our suggested system. The derivation of the bit error probability and the introduction of the notion of users-throughput product for the aforementioned system are given in Section III. In Section IV, we present some numerical results where we investigate the effect of some parameters (the index of overlap, the number of users, the pulse-position multiplicity, etc.) on the performance of the optical OPPM-CDMA system. Comparisons with other systems are also presented in this section. Finally, we give our conclusion in Section V.

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The author is with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 Singapore, on leave from the Department of Electrical Engineering, Faculty of Engineering, University of Alexandria, Alexandria 21544 Egypt.

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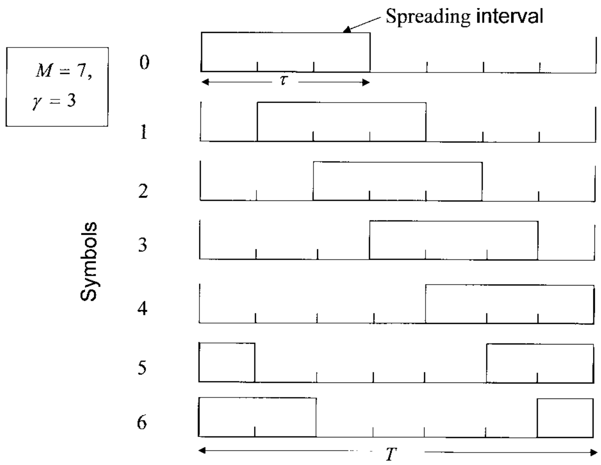

Fig. 1. An example of OPPM signal formats and their spreading intervals of a single user with $M = 7$ and $\gamma = 3$. A laser pulse is signaled at the leading edge of one spreading interval.

II. Optical OPPM-CDMA System Description

A. Optical OPPM Signal Formats

In optical OPPM channel, with multiplicity M and index of overlap $\gamma \in \{1, 2, \cdots, M\}$, the information is conveyed by the position of a laser pulse of duration T_c, called the chip time, within a time frame of width T. Each laser pulse has an allowable spreading interval of duration τ, Fig. 1. This spreading interval will sometimes be called a slot. Each slot is subdivided into γ smaller subintervals of width τ/γ each. An overlap with depth $(1 - (1/\gamma))\tau$ is allowed between any two adjacent spreading intervals. There are M possible positions within the time frame. A transmitted pulse (of duration T_c) is said to be in position $x, x \in \{0, 1, 2, \cdots, M-1\}$, if it is initiated at time $x(\tau/\gamma)$. The corresponding spreading interval starts at the same instant of the laser pulse and ends τ s later, Fig. 1. To simplify the analysis we allow cyclic shifts to occur, that is if for some x it happened that $x(\tau/\gamma) + \tau > T$, the spreading interval is wrapped to the beginning of the frame (cf. Symbols 5 and 6 of Fig. 1.) The number of disjoint slots within a time frame is equal to M/γ. The relation between $T, \gamma, M,$ and τ is thus

$$T = \frac{M}{\gamma}\,\tau, \qquad \gamma \in \{1, 2, \cdots, M\}.$$

It is remarkable that PPM is a special case of OPPM when $\gamma = 1$.

B. Optical OPPM-CDMA System Model

The model for an optical OPPM-CDMA communication system is shown in Fig. 2. The transmitter is composed of N simultaneous information sources or users. Each user transmits M-ary continuous data symbols. The output symbol of the kth information source modulates the position of a tall narrow laser pulse (of width T_c) to form the OPPM initial signal. This signal is then passed to the CDMA encoder where it is spread into w shorter laser pulses with same width T_c according to the signature sequence (of length L and weight w) which characterizes the kth user. The spreading is allowed to occur within the spreading interval only. Thus $\tau = LT_c$. The output waveform is finally transmitted over the optical channel.

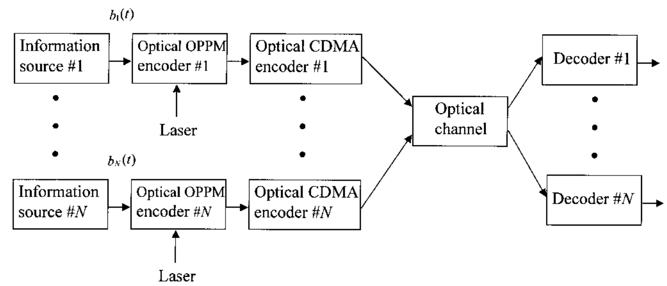

Fig. 2. An optical OPPM-CDMA system model.

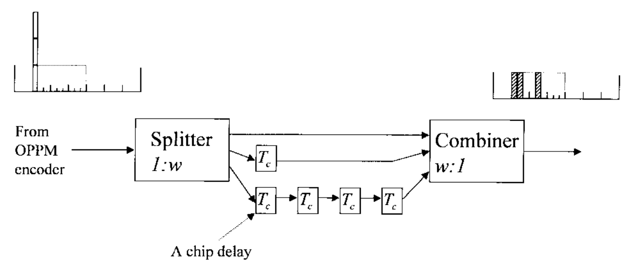

Fig. 3. An example of an optical CDMA encoder for an unwrapped signal. (Symbol 1 of Fig. 1.) A signature code of 110010000 is assumed.

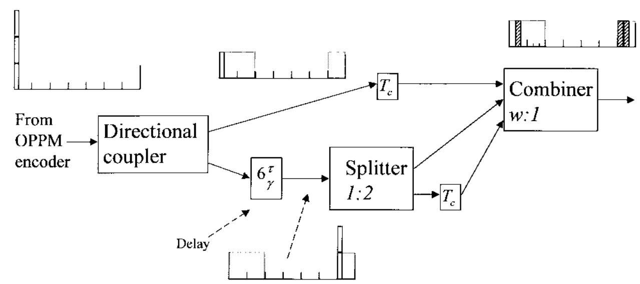

Fig. 4. An example of an optical CDMA encoder for a wrapped signal. (Symbol 6 of Fig. 1.) A signature code of 110010000 is assumed.

A traditional way to achieve optical spreading is to use an optical tapped delay line which is composed of a splitter, delayers, and a combiner, Fig. 3. Wrapped OPPM-CDMA signals need special techniques for generation, Fig. 4. In this case for the spreading sequence to fit properly within the spreading interval, the following condition must be satisfied

$$\frac{\tau}{\gamma} = \text{integer} \times T_c \quad \text{or} \quad \frac{L}{\gamma} = \text{integer}.$$

An example of the transmitted signal formats of a single user is shown in Fig. 5.

At the receiving end, the received optical signal (composed of the sum of N delayed users' optical signals) is correlated with the same signature sequence which characterizes the desired user and then converted (using a photodetector) into an electrical signal which is passed to the OPPM decoder to obtain the data. The correlator can also be an optical tapped delay line matched to the underline code sequence [6], whereas the OPPM decoder is merely a comparison between the photon counts collected over the M time slots: the number of the

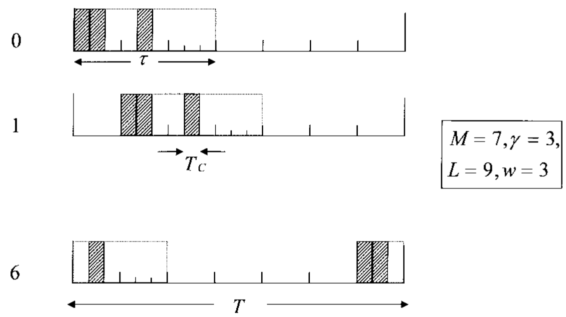

Fig. 5. An example of the transmitted signal formats of a single user in an OPPM-CDMA system with $M = 7, \gamma = 3, L = 9$, and $w = 3$. A signature code of 110010000 is assumed.

slot with the greatest count is declared to be the transmitted symbol.

III. Performance Evaluation of Optical OPPM-CDMA Systems

A. The Decision Rule and the Probability of Error

Each receiver counts the photons collected in the permissible chips (determined by its signature code) of every slot within the time frame. The number of the slot having the largest count is declared to be the transmitted symbol. We denote the photon count collected in slot $i \in \{0, 1, \cdots, M-1\}$ by Y_i. Symbol i is thus declared to be the true one if $Y_i > Y_j$ for every $j \neq i$. Hence, for equiprobable data symbols, the probability of error can be written as

$$P[E] = \sum_{i=0}^{M-1} P[E|i] \Pr\{D = i\} = \frac{1}{M} \sum_{i=0}^{M-1} P[E|i]$$

where D is a random variable that denotes the transmitted data symbol and

$$P[E|i] = \Pr\{Y_j \geq Y_i, \text{ some } j \neq i | D = i\}.$$

It is obvious, because of the symmetry of the channel, that the last probability is independent of i. Hence

$$P[E] = \Pr\{Y_j \geq Y_0, \text{ some } j \neq 0 | D = 0\}. \qquad (1)$$

B. The Probability of Interference

In our derivation of the bit error rate (BER), we assume that all users are synchronized at a frame level. This assumption simplifies the mathematical analysis and gives more insights on the problem under consideration. Removing this assumption would not lead to much differences on the obtained results but only complicated analysis. Indeed this assumption guarantees that each user can interfere with the desired user at only one pulse position or does not interfere at all. If this assumption was removed there would be a rare possibility of interfering at two pulse positions as well. Since this event is very rare, no much differences in the results would be expected. Denote by $\kappa_i, i \in \{0, 1, \cdots, M-1\}$, the number of other users that cause interference to the desired user within slot i. Moreover denote the vector $(\kappa_0, \kappa_1, \cdots, \kappa_{M-1})$ by κ_0^{M-1}. If the probability that a single user causes an interference with the desired user at one pulse position is denoted by p, then the random variable $\kappa_i, i \in \{0, 1, \cdots, M-1\}$, admits a binomial distribution with parameters p and $N-1$

$$\Pr\{\kappa_i = l_i\} = \binom{N-1}{l_i} p^{l_i} (1-p)^{N-1-l_i}$$

where $l_i \in \{0, 1, \cdots, N-1\}$ is a realization for κ_i.

The following proposition gives a characterization of the probability p described above.

Proposition 1: In a frame-level synchronous optical OPPM-CDMA channel employing OOC's with weight $w > 1$, length $L \geq w^2$, and auto- and cross-correlation constraints bounded by one, if p denotes the probability that a single user interferes with the desired user at one pulse position then

$$p = \gamma \frac{w^2}{ML}$$

where M and γ denote the pulse position multiplicity and index of overlap, respectively.

Proof: If the data transmitted by the interfering user is within a similar slot as that transmitted by the desired user, an interference with probability w^2/ML occurs on the average. On the other hand, if the active slot of the interfering user overlaps by $(1 - (|j|/\gamma))\tau, |j| \in \{1, 2, \cdots, \gamma\}$, with that of the desired user, an interference with probability $(w^2/ML)(1 - (|j|/\gamma))$ occurs on the average. Thus

$$p = \sum_{j=-\gamma}^{\gamma} \frac{w^2}{ML} \left(1 - \frac{|j|}{\gamma}\right) = \gamma \frac{w^2}{ML}. \qquad \square$$

C. Bit Error Rate Upper Bound

We now start deriving an upper bound on the probability of error given in (1)

$$\begin{aligned} P[E] &= \Pr\{Y_j \geq Y_0, \text{ some } j \neq 0 | D = 0\} \\ &= \sum_{l_0^{M-1}} \Pr\{Y_j \geq Y_0, \text{ some } j \neq 0 | D = 0 \\ &\qquad \kappa_0^{M-1} = l_0^{M-1}\} \Pr\{\kappa_0^{M-1} = l_0^{M-1}\} \\ &\leq \sum_{l_1^{M-1}} \Pr\{Y_j \geq Y_0, \text{ some } j \neq 0 | D = 0 \\ &\qquad \kappa_0 = 0, \kappa_1^{M-1} = l_1^{M-1}\} \Pr\{\kappa_1^{M-1} = l_1^{M-1}\}. \end{aligned}$$

We further use a union bound, taking into account that the out-of-phase autocorrelation is bounded by one and that at most $\gamma - 1$ hits (self-interference) may occur in other slots due to code pulses in one slot of the desired user

$$\begin{aligned} &\Pr\{Y_j \geq Y_0, \text{ some } j \neq 0 | D = 0, \kappa_0 = 0, \kappa_1^{M-1} = l_1^{M-1}\} \\ &\quad \leq (M - \gamma) \Pr\{Y_1 \geq Y_0 | D = 0 \\ &\qquad \kappa_0 = 0, \kappa_1 = l_1, \nu_1 = 0\} \\ &\qquad + \sum_{j=1}^{\gamma-1} \Pr\{Y_j \geq Y_0 | D = 0, \kappa_0 = 0, \kappa_j = l_j\} \end{aligned}$$

where, for any $j \in \{1, 2, \cdots, M-1\}, \nu_j \in \{0, 1\}$ denotes the number of pulses that cause a hit (self-interference) in slot j due to the signature code pulses sent in slot 0 by the desired user. The first term in the right hand side of the last inequality is due to the $M-1-(\gamma-1)$ slots that do not have self-interference with slot 0, i.e., $\nu_j=0$ with probability 1 for these slots. The second term, however, is due to the remaining $\gamma-1$ slots. These slots interfere with slot 0 at a positive probability, i.e., $\Pr\{\nu_j=1\}>0$. Assuming uniformly distributed marks in the code sequences it is easy to see that $\Pr\{\nu_1=1\}=w(w-1)/(L-1)$ and hence

$$\begin{aligned}\Pr\{Y_j \ge Y_0,\ \text{some } j\ne 0 | D=0, \kappa_0=0, \kappa_1^{M-1}=l_1^{M-1}\}\\ \le (M-\gamma)\Pr\{Y_1\ge Y_0|D=0\\ \kappa_0=0,\kappa_1=l_1,\nu_1=0\}\\ +(\gamma-1)q\Pr\{Y_1\ge Y_0|D=0\\ \kappa_0=0,\kappa_1=l_1,\nu_1=1\}\\ +(\gamma-1)(1-q)\Pr\{Y_1\ge Y_0|D=0\\ \kappa_0=0,\kappa_1=l_1,\nu_1=0\}\end{aligned}$$

where $q \stackrel{\text{def}}{=} \Pr\{\nu_1=1\}=w(w-1)/(L-1)$. Hence

$$P[E]\le\sum_l\left[(M-1)P_1+(\gamma-1)\,\frac{w(w-1)}{L-1}\,(P_2-P_1)\right]\cdot\Pr\{\kappa_1=l\}$$

where

$$\begin{aligned}\Pr\{\kappa_1=l\}&=\binom{N-1}{l}p^l(1-p)^{N-1-l}\\ P_1&=\Pr\{Y_1\ge Y_0|D=0,\kappa_0=0,\kappa_1=l,\nu_1=0\}\\ &=\sum_{y_1=0}^{\infty}e^{-Ql}\frac{(Ql)^{y_1}}{y_1!}\cdot\sum_{y_0=0}^{y_1}e^{-Qw}\frac{(Qw)^{y_0}}{y_0!}\end{aligned}$$

and

$$\begin{aligned}P_2&=\Pr\{Y_1\ge Y_0|D=0,\kappa_0=0,\kappa_1=l,\nu_1=1\}\\ &=\sum_{y_1=0}^{\infty}e^{-Q(l+1)}\,\frac{(Q(l+1))^{y_1}}{y_1!}\,\cdot\sum_{y_0=0}^{y_1}e^{-Qw}\,\frac{(Qw)^{y_0}}{y_0!}.\end{aligned}$$

Here, Q denotes the average number of photons per transmitted pulse. From this analysis, we obtain the required upper bound

$$\begin{aligned}P[E]\le{}&(M-1)\sum_{l=0}^{N-1}\binom{N-1}{l}p^l(1-p)^{N-1-l}\\ &\cdot\sum_{y_1=0}^{\infty}e^{-Ql}\,\frac{(Ql)^{y_1}}{y_1!}\cdot\sum_{y_0=0}^{y_1}e^{-Qw}\,\frac{(Qw)^{y_0}}{y_0!}\\ &+(\gamma-1)\,\frac{w(w-1)}{L-1}\sum_{l=0}^{N-1}\\ &\cdot\binom{N-1}{l}p^l(1-p)^{N-1-l}\\ &\times\sum_{y_1=0}^{\infty}\left[e^{-Q(l+1)}\,\frac{(Q(l+1))^{y_1}}{y_1!}-e^{-Ql}\,\frac{(Ql)^{y_1}}{y_1!}\right]\\ &\cdot\sum_{y_0=0}^{y_1}e^{-Qw}\,\frac{(Qw)^{y_0}}{y_0!}.\end{aligned}\qquad(2)$$

Finally, the bit error rate P_b can be obtained from the relation $P_b=M/2(M-1)P[E]$.

D. Bit Error Rate Lower Bound

To be able to measure the tightness of our previous upper bound we derive in this subsection a lower bound on the probability of error given in (1). A given interfering user can interfere with the desired user at either one or two pulse positions, or does not interfere at all. If it interferes with two pulse positions, the interfering pulses must occur within exactly two different slots of the desired user (one pulse per each slot). In order to provide a lower bound on the error rate we are going to assume (in this subsection only) that the possibility of interference at two pulse positions will never happen. In our previous analysis on the upper bound we did not encounter the situation of two pulse positions since our analysis relied on the interference within one slot only. That is we only considered the marginal distribution of κ_i. In this section, however, we use a joint distribution of both κ_i and κ_j. Keeping this in mind we can now find the required lower bound on the probability of error

$$\begin{aligned}P[E]={}&\Pr\{Y_j\ge Y_0,\ \text{some } j\ne 0|D=0\}\\ \ge{}&\Pr\{Y_j\ge Y_0,\ \text{some } j\ne 0|D=0,\nu_i=0\forall i\ne 0\}\\ \ge{}&(M-1)\Pr\{Y_1\ge Y_0|D=0,\nu_1=0\}\\ &-\binom{M-1}{2}\Pr\{Y_1\ge Y_0,Y_2\ge Y_0|D=0,\\ &\nu_1=\nu_2=0\}.\end{aligned}$$

The first probability in the right-hand side of the last inequality can further be lower bounded as follows:

$$\begin{aligned}\Pr\{Y_1\ge Y_0|D=0,\nu_1=0\}\\ ={}&\sum_{l_0,l_1}\Pr\{\kappa_0=l_0,\kappa_1=l_1\}\\ &\cdot\Pr\{Y_1\ge Y_0|D=0,\nu_1=0,\kappa_0=l_0,\kappa_1=l_1\}\\ \ge{}&\sum_{l_1}\Pr\{\kappa_0=0,\kappa_1=l_1\}\\ &\cdot\Pr\{Y_1\ge Y_0|D=0,\nu_1=0,\kappa_0=0,\kappa_1=l_1\}\end{aligned}$$

where

$$\begin{aligned}\Pr\{Y_1\ge Y_0|D=0,\nu_1=0,\kappa_0=0,\kappa_1=l_1\}\\ =\sum_{y_1=0}^{\infty}e^{-Ql_1}\,\frac{(Ql_1)^{y_1}}{y_1!}\cdot\sum_{y_0=0}^{y_1}e^{-Qw}\,\frac{(Qw)^{y_0}}{y_0!}\end{aligned}$$

and

$$\Pr\{\kappa_0=0,\kappa_1=l_1\}=\binom{N-1}{l_1}P_{01}^{l_1}\cdot P_{00}^{N-1-l_1}.$$

Here $P_{rs},(r,s\in\{0,1\})$, denotes the probability that a single user interferes at exactly s pulse positions in slot 0 and r pulse positions in slot 1 of the desired user. Since $P_{11}=0$, we have

$$\begin{aligned}P_{01}&=p-P_{11}=p\quad\text{and}\\ P_{00}&=(1-p)-P_{01}=1-2p.\end{aligned}$$

This implies that

$$\Pr\{\kappa_0 = 0, \kappa_1 = l_1\} = \binom{N-1}{l_1} p^{l_1}(1-2p)^{N-1-l_1}.$$

The second probability can be upper bounded as follows:

$$\begin{aligned}
&\Pr\{Y_1 \geq Y_0, Y_2 \geq Y_0 | D=0, \nu_1 = \nu_2 = 0\} \\
&\quad \leq \Pr\{Y_1 + Y_2 \geq 2Y_0 | D=0, \nu_1 = \nu_2 = 0\} \\
&\quad = \Pr\{Y' \geq 2Y_0 | D=0, \nu_1 = \nu_2 = 0\} \\
&\quad \leq \Pr\{Y' \geq 2Y_0 | D=0, \nu_1 = \nu_2 = 0, \kappa_0 = 0\} \\
&\quad = \sum_{l'} \Pr\{\kappa' = l'\} \Pr\{Y' \geq 2Y_0 | D = 0 \\
&\qquad\qquad \nu_1 = \nu_2 = 0, \kappa_0 = 0, \kappa' = l'\}
\end{aligned}$$

where $Y' = Y_1 + Y_2$ and $\kappa' = \kappa_1 + \kappa_2$. Here κ' denotes the number of other users that cause interference to the desired user within both slots 1 and 2. κ' is again a binomial random variable but with parameter p' and $N-1$. Noticing that $\nu_1 = \nu_2 = 0$ and following the proof of Proposition 1 with slight modification we can check that $p' = 2p$. Thus

$$\Pr\{\kappa' = l'\} = \binom{N-1}{l'} (2p)^{l'}(1-2p)^{N-1-l'}.$$

Making use of Chernoff bound, we get for any $z \geq 1$

$$\begin{aligned}
&\Pr\{Y' \geq 2Y_0 | D=0, \nu_1 = \nu_2 = 0, \kappa_0 = 0, \kappa' = l'\} \\
&\quad \leq E[z^{Y'-2Y_0} | D=0, \nu_1 = \nu_2 = 0, \kappa_0 = 0, \kappa' = l'] \\
&\quad = e^{Ql'(z-1)} \cdot e^{-Qw(1-z^{-2})}.
\end{aligned}$$

Choosing a value of z so as to minimize the right-hand side yields $z_0 = (2w/l')^{1/3}$. Thus

$$\begin{aligned}
&\Pr\{Y' \geq 2Y_0 | D=0, \nu_1 = \nu_2 = 0, \kappa_0 = 0, \kappa' = l'\} \\
&\quad \leq \begin{cases} \exp[Ql'(z_0-1) - Qw(1-z_0^{-2})]; & \text{if } l' < 2w, \\ 1; & \text{otherwise.} \end{cases}
\end{aligned}$$

From the previous analysis we get the required lower bound

$$\begin{aligned}
P[E] \geq{}& (M-1) \sum_{l=0}^{N-1} \binom{N-1}{l} p^l (1-2p)^{N-1-l} \\
&\cdot \sum_{y_1=0}^{\infty} e^{-Ql} \frac{(Ql)^{y_1}}{y_1!} \cdot \sum_{y_0=0}^{y_1} e^{-Qw} \frac{(Qw)^{y_0}}{y_0!} \\
&- \binom{M-1}{2} \sum_{l=0}^{2w-1} \binom{N-1}{l} \\
&\cdot (2p)^l (1-2p)^{N-1-l} \cdot e^{Ql(z_0-1)} \cdot e^{-Qw(1-z_0^{-2})} \\
&- \binom{M-1}{2} \sum_{l=2w}^{N-1} \binom{N-1}{l} \\
&\cdot (2p)^l (1-2p)^{N-1-l}
\end{aligned} \tag{3}$$

where $p = \gamma(w^2/ML)$ and $z_0 = (2w/l)^{1/3}$.

E. Users-Throughput Product

One important parameter of performance evaluation in practice is the data rate. For a given user, the rate of data transmission (throughput) is given by the amount of information transmitted per second by this user

$$R_T \triangleq \frac{\log M}{T} = \frac{\gamma \log M}{M\tau} = \frac{\gamma \log M}{MLT_c} \quad \text{nats/s.}$$

Here the natural number e is taken as the basis of the "log" function. For the sake of convenience (since the pulsewidth is always fixed), we define the throughput-pulsewidth product

$$R_0 \triangleq R_T T_c = \frac{\gamma \log M}{ML} \quad \text{nats/chip.} \tag{4}$$

Moreover, we define the users-throughput product (denoted by NR) as the product of the number of users times R_0

$$\text{NR} \triangleq N \cdot R_0 = N \cdot \frac{\gamma \log M}{ML} \quad \text{nats/chip.}$$

NR is a measure of the total data rate (from all users) transmitted within the channel. In our performance evaluation we are interested in characterizing the maximum throughput (of the OPPM-CDMA channel) that can be achieved while keeping the bit error rate below a prescribed threshold. In other words, we will allow the parameters γ, M, and L to vary so as to maximize the throughput under the constraint that $P_b \leq \epsilon$, some $\epsilon > 0$. Hence, we define $R_{0,\max}$ and $\text{NR}_{\max}$ as follows:

$$R_{0,\max} \triangleq \max_{\substack{\gamma, M, L: \\ P_b \leq \epsilon}} R_0, \quad \text{NR}_{\max} \triangleq \max_{\substack{\gamma, M, L: \\ P_b \leq \epsilon}} \text{NR}. \tag{5}$$

IV. Numerical Results

A. Bit Error Rate (BER)

In our numerical calculations, we hold both the rate of data transmission R_T and pulsewidth T_c fixed. Thus their product R_0 is also fixed. In this section, we will evaluate the BER performance of the above system under an average energy per bit constraint. Thus, if we denote by μ the number of transmitted photons per information nat, then Q in (2) and (3) can be written as $Q = (\mu \log M/w)$. Fig. 6 shows the BER versus the average photons per nat for a fixed number of users ($N = 100$) and a constraint on the throughput-pulsewidth product ($R_0 = 10^{-4}$). Different values of γ are assumed in this figure and M is set to 32. The code length is chosen so as to satisfy the constraint on throughput-pulsewidth product, whereas maximum code weight is chosen so as to satisfy the constraint on OOC's [1]

$$N \leq \frac{L-1}{w(w-1)}.$$

The performance of OOK-CDMA is also superimposed on the same figure. The analysis of this system can be found in [7].

As seen from the figure, when γ increases, the system performance improves significantly. As an example, for a system with signal energy of 65 photons/nat, the bit error rate equals 5.62×10^{-4} for OOK-CDMA and is upper bounded by 2.24×10^{-5} for PPM-CDMA, 1.24×10^{-8} for OPPM-CDMA with $\gamma = 4$, and 1.73×10^{-11} for OPPM-CDMA with

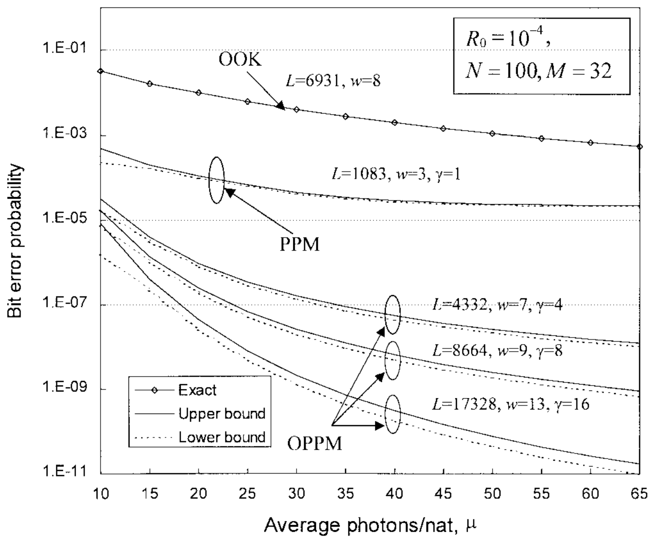

Fig. 6. A comparison between the bit error rates of optical OOK-, PPM, and OPPM-CDMA receivers (with $M = 32$ and different values of γ for OPPM) under the constraints of fixed throughput, chip time, and number of users.

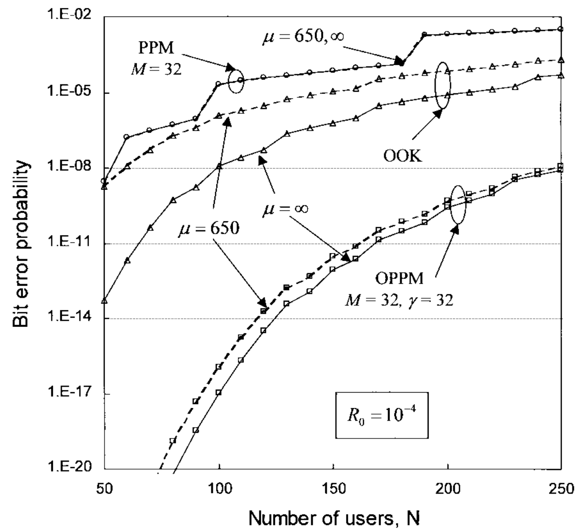

Fig. 7. A comparison between the bit error rates of optical OOK-, PPM-, and OPPM-CDMA receivers versus the number of users when $\mu \in \{650, \infty\}, R_0 = 10^{-4}$.

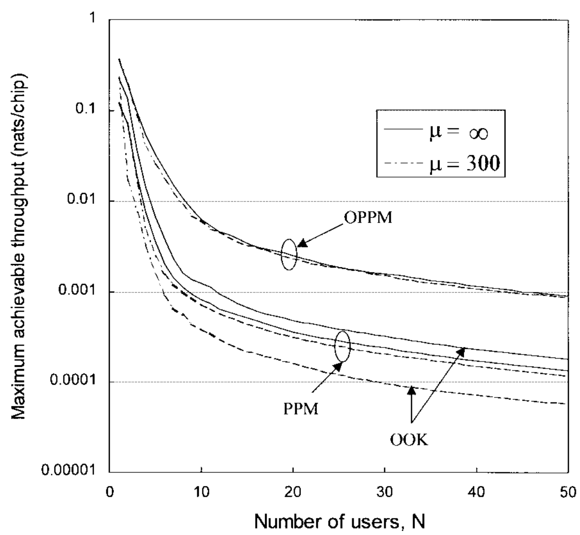

Fig. 8. A comparison between the maximum achievable throughput of optical OOK-, PPM-, and OPPM-CDMA receivers versus the number of users when $\mu \in \{300, \infty\}$ and the BER does not exceed 10^{-9}.

$\gamma = 16$. It is also obvious that if the bit error rate is required not to exceed 10^{-9}, then both OOK- and PPM-CDMA cannot be used with a 100 users even if the average energy is increased without limit. Rather, OPPM-CDMA with suitable indices of overlap can be used with finite energy in a reliable way.

To demonstrate the tightness of our upper bound, lower bounds on the bit error rates for both PPM- and OPPM-CDMA are also plotted in Fig. 6. It is clear that the lower bounds are very close to the upper bounds especially for small values of γ. The code lengths and weights that achieve the aforementioned constraints are further indicated in the same figure. It is obvious that the code length L is directly proportional to γ [cf., (4)]. For $\gamma = 4$, the required code length for OPPM-CDMA is less than that required by OOK-CDMA and for $\gamma = 8$ the code length of OPPM-CDMA is about 1.25 times that of the OOK-CDMA. Actually, they are related as follows:

$$L_{\text{OPPM}} = \frac{\gamma \log_2 M}{M} L_{\text{OOK}}.$$

In Fig. 7, we plot the error probabilities versus the number of users when $\mu = 650$ and R_0 is still constrained as above. The limits of the error probabilities as μ approaches infinity are also superimposed in the same graph for the sake of convenience. We can notice from the figure that in this case OOK-CDMA outperforms PPM-CDMA but OPPM-CDMA is still the best. This demonstrates that when the constraint on the average energy is relaxed OOK becomes better than PPM but not than OPPM. Moreover, it is obvious that a small amount of energy is sufficient for OPPM-CDMA error probability to become very close to its limit. However, for OOK-CDMA a much more greater energy is required for the error probability to become close to its limit.

B. Maximum Achievable Throughput

In this section, we evaluate both $R_{0,\max}$ and $\text{NR}_{\max}$ for a given threshold of $\epsilon = 10^{-9}$. We have performed the required calculations as given by (5). Our results are plotted (versus N) in Figs. 8 and 9 when $\mu \in \{300, 500, \infty\}$. The corresponding results for both OOK- and PPM-CDMA are also plotted on the same figures for the sake of comparison. From Fig. 8, we can see that the throughput improves significantly when using OPPM rather than OOK (a log scale is used.) It is also seen that OOK is slightly better than PPM only for large values of the average energy. The difference between the values of $R_{0,\max}$ when $\mu = 300$ and $\mu = \infty$ is negligible for both PPM- and OPPM-CDMA. However, this difference is significant for OOK-CDMA which indicates the potential improvement when using OPPM-CDMA. In Fig. 9, the $\text{NR}_{\max}$ of both PPM- and OPPM-CDMA are calculated when $\mu = 500$ but that of OOK-CDMA is evaluated when $\mu = \infty$. Of course, this comparison is biased in favor of OOK-CDMA. Our aim is to illuminate the potential advantage of OPPM-CDMA as an

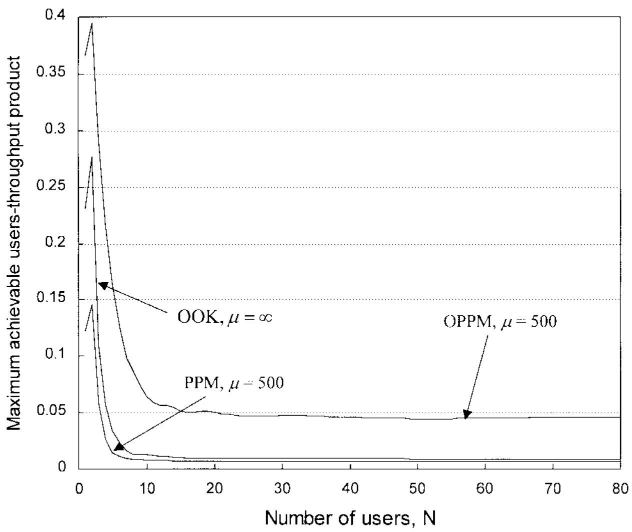

Fig. 9. A comparison between the maximum achievable users-throughput products of optical OOK-, PPM-, and OPPM-CDMA receivers versus the number of users when $\mu \in \{500, \infty\}$ and the BER does not exceed 10^{-9}.

alternative of OOK-CDMA. From this figure we can extract two interesting issues. First, the maximum achievable users-throughput product is almost constant for all values of N greater than ten. In this case, the $\text{NR}_{\max}$ of OPPM-CDMA is about five times greater than that of OOK-CDMA. We can thus consider the users-throughput product as a figure of merit for this type of channels. The second issue arises if the number of users is low. In this case the users-throughput product increases as the number of users decreases and at $N = 8$ the value of $\text{NR}_{\max}$ for OPPM-CDMA becomes almost 6.7 times greater than that for OOK-CDMA.

V. Concluding Remarks

Direct-detection optical code-division multiple-access (CDMA) systems employing overlapping pulse-position modulation (OPPM) schemes have been proposed. Both lower and upper bound on the bit error rate have been derived for this system taking into account the photodetector's Poisson shot noise and the multiple-user interference. Both fixed data rate and fixed pulsewidth have been assumed in our numerical evaluation of the BER. For a given number of users, the best parameters of an optical orthogonal code have been chosen so as to satisfy the previous requirements. The notion of users-throughput product has been introduced as a measure to the total data rate that can be accommodated within the entire channel while keeping the error rate below a prescribed parameter. The performance of our channel (in terms of bit error rate, maximum achievable throughput, and maximum achievable users-throughput product) has been compared to traditional optical CDMA systems, namely OOK- and PPM-CDMA. Our results reveal significant improvement in the performance when using OPPM-CDMA. We can extract the following concluding remarks.

1) OPPM-CDMA is more efficient and has a much more better error rate than that of both OOK- and PPM-CDMA.
2) OPPM-CDMA can tolerate the throughput and users capacity limitations of the traditional optical CDMA systems. Indeed, the maximum throughput that can be achieved per each user in the OPPM-CDMA channel is greater by almost five times than that of OOK-CDMA (for most of the time) and about 10 times than that of PPM-CDMA. For small number of users the OPPM-CDMA throughput can further be increased to about 6.7 times greater than that of OOK-CDMA.
3) The maximum achievable users-throughput product is almost a constant quantity and can be considered as a figure of merit when designing this type of systems.
4) The implementation of the OPPM-CDMA system is rather simple. The transmitter involves only time delaying of the optical pulses and the receiver (unlike OOK-CDMA) does not require knowledge of the signal or noise power. Furthermore, the required code length of OPPM-CDMA is less than that of OOK-CDMA as long as $\gamma \leq M/\log_2 M$. The complexity of this system arises only when we choose large values of M and γ.

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Hossam M. H. Shalaby (S'83–M'91) was born in Giza, Egypt, in 1961. He received the B.S. and M.S. degrees in communications from the University of Alexandria, Egypt, in 1983 and 1986, respectively, and the Ph.D. degree from the University of Maryland, College Park, in 1991, all in electrical engineering.

In 1991, he joined the Department of Electrical Engineering, the University of Alexandria, Egypt, as an Assistant Professor. He was promoted to Associate Professor in 1996. From March to April 1996, he was a Visiting Professor at the Electrical Engineering Department, Beirut Arab University, Lebanon. He has been on leave from the University of Alexandria since September 1996, where he joined two places: From September 1996 to January 1998 he was an Associate Professor with the Electrical and Computer Engineering Department, International Islamic University Malaysia. From February 1998 to December 1998, he was with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, as a Senior Lecturer and then as an Associate Professor (since January 1999). His research interests include optical communications, optical CDMA, spread-spectrum communications, and information theory.

Dr. Shalaby was a recipient of an SRC fellowship from 1987 to 1991 (Systems Research Center, MD), and both a State Award and Soliman Abd-El-Hay Award in 1995 (Academy of Scientific Research and Technology, Egypt). He served as a Chairman of the Student Activities Committee of Alexandria IEEE Subsection from 1995 to 1996. He has also served as a technical referee for IEEE TRANSACTIONS ON COMMUNICATIONS, IEEE TRANSACTIONS ON INFORMATION THEORY, and IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS. He is listed in the 14th edition of *Marquis Who's Who in the World*, 1997.