Average SER of MPPM Technique over Exponentiated Weibull Fading FSO Channels Considering Fog and Beam Divergence

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Abstract
The performance of free-space optical (FSO) transmission systems adopting multi-pulse pulse-position modulation (MPPM) scheme is investigated taking into account the effects of the atmospheric turbulence, fog, and beam divergence. The atmospheric turbulence is modeled by the exponentiated Weibull (EW) distribution. A closed-form expression for the symbol-error rate (SER) of our MPPM-FSO system is obtained based on MeijerG function. The results show that the turbulence effect can be mitigated by aperture averaging.

Keywords: Beam divergence, exponentiated Weibull channels, fog, free-space optics (FSO), multi-pulse pulse-position modulation (MPPM).

1. Introduction

The importance of free-space optical (FSO) communication has increased recently because it is a very promising way of providing high speed, large capacity, and cost effective wireless data transmission [1]. However, the performance of FSO communication systems is highly affected by atmospheric conditions. One of the main phenomena that degrades the performance of FSO communication systems is atmospheric turbulence or scintillation, which results from inhomogeneities in the temperature and pressure of the atmosphere. It leads to random fluctuations in both the amplitude and phase of the received signal, due to variations in the refractive index, which degrade the system performance [2].

Several statistical distributions have been assumed to characterize the atmospheric turbulence in the literature. The most widely accepted distributions are the Log-Normal (LN), Gamma-Gamma (GG), and Exponentiated Weibull (EW) models. The last one provides a better fit to the experimental and simulation data of the channel fading when compared to both LN and GG models under all of the aperture averaging conditions [3]. Therefore, EW distribution is used to describe the atmospheric turbulence effects on FSO transmission in this paper.

The performance of FSO system adopting MPPM under Gamma-Gamma fading channels has been investigated in [2]. In [4], a new closed-form expression for average BER of OOK modulation technique has been derived using MeijerG function. In [5], approximate expressions for the BER and outage probability of BPSK and MPSK modulation techniques in the EW distribution based FSO systems have been derived. Although in [6] approximate expressions for symbol-error rate (SER) of MPPM under EW fading distribution have been derived based on generalized Gauss-Laguerre quadrature function, the effects of both fog and beam divergence have not been considered. The main contribution in this paper is deriving exact expression for the average SER of MPPM over EW distribution considering both fog and beam divergence. The obtained expression is used to investigate effects of aperture averaging on the SER performance under moderate and strong turbulence conditions in FSO transmission channel.

The rest of this paper is organized as follows. In Section 2, mathematical model for FSO adopting MPPM is explored. In Section 3, exponentiated Weibull fading distribution is presented. Sections 4 is devoted for analyzing the performance FSO transmission system considering attenuations due to both fog and beam divergence. In Section 5, the performance of FSO system adopting MPPM is investigated under different weather conditions and fading levels. Finally, the conclusions are given in Section 6.

2. FSO-MPPM System Model

In MPPM modulation techniques, a symbol duration, T, is divided into N time-slots, each has a duration $\tau_s = T/N$. Optical power illuminates $w < N$ signal time-slots. The MPPM symbol for any number of time-slots $N \geq 1$ and any $w \in \{1,2,\cdots,N\}$ is selected from the set [7]:

$$S_{MPPM} \stackrel{\text{def}}{=} \left\{ \mathbf{B} \in \{0,1\}^N : \sum_{i=1}^{N} B_i = w \right\}. \tag{1}$$

The cardinality of this set is $\binom{N}{w}$ and the number of bits per MPPM symbol is $\left\lfloor \log_2 \binom{N}{w} \right\rfloor$, where $\lfloor x \rfloor$ is the maximum integer less than x. The bit-rate for system adopting the MPPM technique is $R_b = \left\lfloor \log_2 \binom{N}{w} \right\rfloor / T$. At the receiver side, the photodiode (PD) converts the received optical intensity variations into corresponding variations in the electrical domain. The output current of the PD can be written as:

$$y(t) = I_{ph}(t) \sum_{i=0}^{N-1} B_i(t) \, \text{rect}(t - i\tau_s) + n(t), \tag{2}$$

where $I_{ph}(t) = \mathscr{R} \binom{N}{w} h_t(t) \hat{P}$, $\mathscr{R}$ is the responsivity of the photodiode, $\hat{P}$ is the average launched power, $n(t)$ is Gaussian noise with variance σ_n^2, $h_t(t) = \xi h$ is the overall channel gain, and h is turbulence gain. $\xi = \xi(L)/\xi_d(L)$ is the normalized path loss coefficient with $\xi_d(L)$ being the path loss of direct link in clear-weather conditions and $\xi(L)$ can be calculated by combining weather attenuation with geometric losses as $\xi(L) = 10^{-UL/10}(D_R^2/(D_T + \theta_T L)^2)$, where D_R and D_T are the receiver and transmitter aperture diameters, U is the weather dependent attenuation coefficient (in dB/km), L is the FSO transmission path distance, and θ_T is the optical beam divergence angle (in mrad) [8–10]. The values of U for different weather conditions are provided in [11]. Furthermore, the instantaneous electrical signal-to-noise ratio (γ) can be expressed as [12]:

$$\gamma = \frac{\left[\mathscr{R} \binom{N}{w} h_t(t) \hat{P} \right]^2}{\sigma_n^2} = \hat{\gamma} h_t^2(t), \tag{3}$$

where $\hat{\gamma}$ is the average electrical signal-to-noise ratio.

3. Exponentiated Weibull Fading FSO Channel Model

The effect of turbulence gain h is well described by the exponentiated Weibull (EW) distribution [3] with a probability density function (pdf) given as:

$$f_{EW}(h;\beta,\eta,\alpha) = \frac{\alpha\beta}{\eta} \left(\frac{h}{\eta} \right)^{\beta-1} \exp\left[-\left(\frac{h}{\eta} \right)^{\beta} \right] \left\{ 1 - \exp\left[-\left(\frac{h}{\eta} \right)^{\beta} \right] \right\}^{\alpha-1}, \tag{4}$$

where $\beta > 0$ is a shape parameter related to the scintillation index, $\eta > 0$ is a scale parameter that depends on β, and $\alpha > 0$ is an extra shape parameter that is strongly dependent on the receiver aperture size as follows:

$$\alpha \approx 3.931 \left(\frac{D_R}{\rho_o} \right)^{-0.519}, \quad \beta \approx \left(\alpha \sigma_I^2 \right)^{-6/11}, \quad \eta = \frac{1}{\alpha \Gamma(1 + 1/\beta) g(\alpha,\beta)}, \tag{5}$$

where $\rho_o = \left(1.46 C_n^2 (2\pi/\lambda)^2 L \right)^{-3/5}$ is the atmospheric coherence radius, λ is the optical wavelength, C_n^2 is the refractive index structure constant, σ_I^2 is the scintillation index, $\Gamma(\cdot)$ is the gamma function, and $g(\alpha,\beta)$ is given by:

$$g(\alpha,\beta) = \sum_{j=0}^{\infty} \frac{(-1)^j (j+1)^{-(1+\beta)/\beta} \Gamma(\alpha)}{j! \Gamma(\alpha - j)}. \tag{6}$$

4. Performance Analysis Considering Fading, Fog, and Beam Divergence

The SER of MPPM technique, SER_{MPPM}, is given as [2]:

$$\text{SER}_{\text{MPPM}}(\gamma) \leq \frac{\binom{N}{w} - 1}{2} \, \text{erfc}\left(\sqrt{\frac{\gamma \log_2 \binom{N}{w}}{4N}} \right). \tag{7}$$

The average SER is obtained by getting the average of $\mathrm{SER_{MPPM}}(\gamma)$ with respect to γ. In order to perform this averaging, we transform the random variable in (4) to get the pdf of the instantaneous signal-to-noise ratio, γ, as follows:

$$f_{EW}(\gamma;\beta,\eta,\alpha) = \frac{\alpha\beta}{2\gamma}\left(\frac{\gamma}{\hat{\gamma}\eta^2\xi^2}\right)^{(\beta/2)} \exp\left[-\left(\frac{\gamma}{\hat{\gamma}\eta^2\xi^2}\right)^{\beta/2}\right]\left[1-\exp\left(-\left(\frac{\gamma}{\hat{\gamma}\eta^2\xi^2}\right)^{\beta/2}\right)\right]^{(\alpha-1)}. \quad (8)$$

Using (7) and (8), the average of $\mathrm{SER_{MPPM}}$ can be evaluated as follows:

$$\overline{\mathrm{SER}}_{\mathrm{MPPM}} = -\int_0^\infty \mathrm{SER_{MPPM}}(\gamma) f_{EW}(\gamma;\beta,\eta,\alpha)d\gamma, \quad (9)$$

$$\overline{\mathrm{SER}}_{\mathrm{MPPM}} \le \left(\binom{N}{w}-1\right)\frac{\alpha\beta}{4(\hat{\gamma}\eta^2\xi^2)^{\beta/2}}\int_0^\infty \gamma^{(\beta/2-1)} \times \mathrm{erfc}\left(\sqrt{\frac{\gamma\log_2\binom{N}{w}}{4N}}\right)\exp\left(-\left(\frac{\gamma}{\hat{\gamma}\eta^2\xi^2}\right)^{\beta/2}\right)\left\{1-\exp\left(-\left(\frac{\gamma}{\hat{\gamma}\eta^2\xi^2}\right)^{\beta/2}\right)\right\}^{(\alpha-1)} d\gamma. \quad (10)$$

Using Newton's generalized binomial expansion, $(1+z)^r = \sum_{j=0}^{\infty}\frac{\Gamma(r+1)z^j}{\Gamma(r-j+1)j!}$ and expressing the exponential and erfc functions in terms of the MeijerG function [13], the $\overline{\mathrm{SER}}_{\mathrm{MPPM}}$ can be written as:

$$\overline{\mathrm{SER}}_{\mathrm{MPPM}} \le \left(\binom{N}{w}-1\right)\frac{\alpha\beta\Gamma(\alpha)}{4\sqrt{\pi}(\hat{\gamma}\eta^2\xi^2)^{\beta/2}} \times \sum_{j=0}^{\infty}\frac{(-1)^j}{j!\Gamma(\alpha-j)}\int_0^\infty \gamma^{(\beta/2-1)} \mathrm{G}_{1,2}^{2,0}\left(\frac{\gamma\log_2\binom{N}{w}}{4N}\middle|_{0,\ 0.5}^{1}\right)\mathrm{G}_{0,1}^{1,0}\left((1+j)\left(\frac{\gamma}{\hat{\gamma}\eta^2\xi^2}\right)^{\beta/2}\middle|_{0}^{-}\right)d\gamma. \quad (11)$$

Using the general integration-form in [13], the average $\mathrm{SER_{MPPM}}$ is given as:

$$\overline{\mathrm{SER}}_{\mathrm{MPPM}} \le \left(\binom{N}{w}-1\right)\left(\frac{4N(w/N)^2}{\hat{\gamma}\log_2\binom{N}{w}\eta^2\xi^2}\right)^{\beta/2}\frac{\alpha\beta\Gamma(\alpha)k^{0.5}l^{(\beta/2-1)}}{4\sqrt{\pi}(2\pi)^{0.5(l+k)-1}} \times \sum_{j=0}^{\infty}\frac{(-1)^j}{j!\Gamma(\alpha-j)}\mathrm{G}_{2l,k+l}^{k,2l}\left(\left(\frac{1+j}{k}\right)^k\left(\frac{4(w/N)^2Nl}{\log_2\binom{N}{w}\hat{\gamma}\eta^2\xi^2}\right)^l\middle|_{\Delta(k,0),\ \Delta(l,-\beta/2)}^{\Delta(l,1-\beta/2),\ \Delta(l,0.5-\beta/2)}\right), \quad (12)$$

where l and k are integers with $\frac{l}{k}=\frac{\beta}{2}$ and $\Delta(b,a)\in\left\{\frac{a}{b},\frac{a+1}{b},\cdots,\frac{a+b-1}{b}\right\}$.

5. Discussion and Numerical Results

In this section, we investigate the performance of MPPM-FSO transmission systems using the obtained SER expression. In Fig. 1a, the effects of turbulent weather and fog, considering beam divergence have

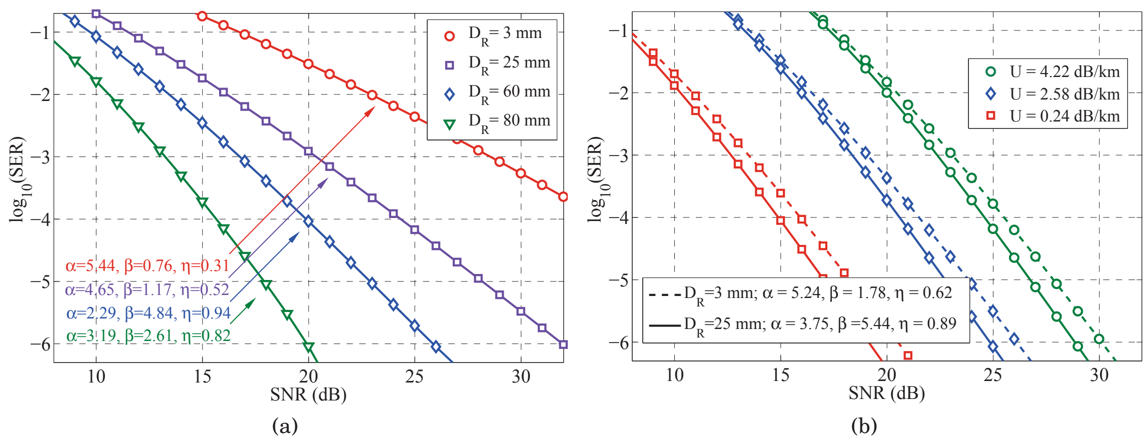

Fig. 1. Average symbol-error rates (SER) for MPPM with ($N=8$, $w=4$) versus the average electrical signal-to-noise ratio SNR (in dB) with optical beam divergence angle $\theta_T = 2\,\mathrm{mrad}$ under: (*a*) moderate turbulence with different aperture sizes, (*b*) weak turbulence with different fog levels..

been investigated at different aperture sizes. The figure shows the variation of the SER versus the SNR for a FSO system with $L = 1220$ m, over moderate turbulence channel, $C_n^2 = 2.1 \times 10^{14}$ m$^{2/3}$, clear weather conditions, $U = 0.19$ dB/km, and optical beam divergence angle $\theta_T = 2$ mrad for different aperture sizes D_R. The values of α, β, and η have been extracted from the best pdf fitting our parameters in [3]. It is clear that the larger the receiver apertures, the better the SER system performance. Because increasing aperture size improves system performance through averaging the effect of turbulence over the aperture area. Specifically, at average SER = 10^{-3}, the SERs performance for $D_R = 25$, 60, and 80 mm outperform that for point receiver, $D_R = 3$ mm, of about 8, 13, and 17 dB, respectively. Thus, it is clear that the aperture-averaging provides an attractive solution to mitigate the turbulence effect on FSO transmission systems adopting MPPM techniques.

Figure 1b illustrates the effects of weak turbulent weather at different fog levels in an EW fading channel, considering beam divergence. The average SERs of MPPM-FSO system are drawn versus the average electrical signal-to-noise ratio over weak turbulent channel, considering different levels of fog, clear, haze, and thin fog ($U = 0.24$, 2.58, and 4.22 dB, respectively). Furthermore, we have considered point receivers with $D_R = 3$ and 25 mm. The performance of receiver with larger aperture size outperforms that of the point receiver by 1.5 dB at SER of 10^{-6} for different fog levels.

6. Conclusion

The performance of FSO transmission systems adopting MPPM techniques have been evaluated. A closed-form expression for symbol-error rate SER for MPPM technique has been derived under EW fading channel taking in account the effects of both fog and beam divergence. The results reveal that, aperture averaging can be used to mitigate both the turbulence effect and fog attenuation on FSO transmission system.

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