Upper and Lower Bounds of Burst Loss Probability for a Core Node in an Optical Burst Switched Network with Pareto distributed arrivals

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Abstract—**A mathematical model is introduced for performance evaluation of an OBS core node employing either no or full wavelength conversion strategies. Furthermore, the model assumes long-range dependent (LRD) traffic arrivals to the OBS intermediate node which are accurately modeled by Pareto distribution. In our proposed model, each output port is imitated by a *GI/M/w/w* queue for which a single performance measure; namely the burst loss probability, is evaluated from the model equations. Also, results of this model are compared with those obtained while assuming short-range dependent (SRD) Poisson arrivals to the core node in the two cases of no and full wavelength conversion. Finally, results show that traditional Poisson traffic models yields over-optimistic performance measures in terms of lower burst loss probability when compared to the more accurate long-range dependent Pareto traffic model. This discrepancy between the two models is much clearer for light traffic scenarios due to the more significant impact of self-similarity.**

Keywords-Optical Burst Switching (OBS); Long-range dependency (LRD); Pareto distribution; Short-range dependency (SRD); Poisson distribution; self-similarity.

I. INTRODUCTION

The tremendous increase of the data rate demand necessitates utilizing the vast bandwidth available on optical fiber links which makes it obligatory to realize the dream of all optical networks (AONs). One of the approaches that target this goal is a paradigm called optical burst switching (OBS) which was first proposed in literature by Qiao and Yoo in [1] and [2]. In OBS, switching is made on a burst by burst basis where the burst comprises of a group of aggregated packets having the same destination and class.

The OBS network architecture consists of three components; ingress nodes, core nodes and egress nodes [3]. The data burst (DB) enters the network through the ingress node after aggregating the data packets with the appropriate assembly algorithm. While at the core node, the CP is processed reserving appropriate resources for the upcoming DB and configuring the switch fabric to bypass the DB upon its arrival to the destined port. The egress node is the destination node at which the DB is disassembled into original packets, each of which is directed to its own destination.

In order to closely simulate the real network scenario thereby verify the effectiveness of protocol designs, it is necessary to model the traffic flows carried over realistic networks. The confidence of the results obtained of mathematical models built for performance evaluation of realistic networks depends on the closeness of the traffic model adopted to the real traffic scenario. For that reason, while building a mathematical model for performance evaluation of an OBS core node, we should model the traffic arrivals to the OBS intermediate node in the most possible accurate form.

Statistical analysis of high-resolution traffic measurements from a wide range of working packet networks, such as Internet, have convincingly shown that the actual traffic streams in such networks exhibit the property of self-similarity or long range dependency (LRD) [4]. That means that similar statistical patterns may occur over different time scales that can vary by many orders of magnitude (i.e. ranging from milliseconds to minutes and even hours). This means that the behavior of these traffic streams significantly departs from the traditional telephone traffic and its related Markov models with short-range dependency (SRD). In particular, the common Poisson arrival process and corresponding analysis based on Erlang-B formula are no longer valid. Alternatively, another probability distribution function rather the conventional Poisson distribution is needed to model these new statistical properties. The Pareto probability distribution has been suggested, as a heavy-tailed distribution, many times as a good fit for such LRD data streams.

The aim of this paper is to present a mathematical model that evaluates an upper and lower bounds of the burst loss probability for DB arrivals at an OBS core node in case of full and no wavelength conversion (NWC and FWC) respectively. Furthermore, DB inter-arrival times are assumed to follow the Pareto distribution in order to accurately model the real self-similar traffic streams in OBSNs. Finally, results of our model that assumes Pareto LRD DB arrivals are compared against traditional models that assume Poisson SRD DB arrivals. Comparison shows that conventional Poisson traffic models gives lower estimates for the burst loss probability when compared to the more realistic Pareto traffic models especially in case of light traffic scenarios.

The remainder of this paper is organized as follows. In Section II, we present a detailed description for our proposed mathematical model. Section III is devoted for the numerical results. Finally, we give our conclusion in Section IV.

II. PROPOSED MODEL DESCRIPTION

This section is organized as follows. First, we give the assumptions made in order to build the model. Next, we the present our model equations for both cases considered: FWC and NWC.

A. Model assumptions

We are going to build our model upon the following set of assumptions:

- We assume that the destination output port for an incoming DB to the OBS core node is uniformly distributed among all available output ports. Thus, it is sufficient to model the behavior of a single output port instead of considering all output ports of the node.
- Each OBS core node considered in our model is assumed to have the following resources:
  - i. A number of w wavelengths available to serve the incoming burst arrivals.
  - ii. No fiber delay lines, i.e. there are no buffering capabilities for contention resolution in the OBS nodes.
  - iii. A number of wavelength converters, each of them can convert the wavelength of the incoming burst to any other free wavelength from the set of the available wavelengths w whenever a contention is encountered by the arriving burst. Typically, the set of available wavelengths is denoted by $\Lambda \stackrel{\text{def}}{=} \{\lambda_1, \lambda_2, \ldots, \lambda_w\}$ while the node has u wavelength converters where $u \in \{1,2,\ldots,w\}$. This means that only u wavelengths of Λ can be converted to any other wavelength in the set, while the remaining w-u wavelengths are nonconvertible ones. We define the node conversion capability as $\gamma \stackrel{\text{def}}{=} \frac{u}{w}$. If $\gamma = 0$, this means that the node has no wavelength conversion capability (NWC), whereas if $\gamma = 1$, this implies that it has full conversion capability (FWC). If $0 < \gamma < 1$, the node has partial wavelength conversion capability (PWC). In this model, we are only considering the two limiting cases of $\gamma = 0$ and $\gamma = 1$, i.e. the calculated burst loss probability from the model can be considered as lower and upper bounds for the burst loss probability that can be achieved when PWC is employed in the OBS core node where the lower bound is set by the case of FWC ($\gamma = 1$) and the upper bound by the case of NWC ($\gamma = 0$).
- Inter-arrival times between incoming DBs to the OBS core node are assumed to follow a Pareto distribution with an average $1/\lambda$ seconds, where λ is the average arrival rate in bursts/second. The service time of an incoming burst is assumed to have an exponential distribution with a mean $1/\mu$ seconds which is equal to the average duration of the data burst.

The Pareto distribution is given by the following probability distribution function (pdf):

$$f_X(x) = \frac{ab^a}{(b+x)^{a+1}} \quad \text{where } x > 0, a > 0 \text{ and } b > 0 \qquad (1)$$

It can be easily shown that the Pareto pdf has a finite mean and infinite variance for $1 < a < 2$ and its mean is given by:

$$E(X) = \frac{b}{a-1} \qquad (2)$$

In order to model a self-similar or LRD traffic stream, we are going to assume that the inter-arrival times are Pareto distributed with parameter a where $1 < a < 2$. This is to make the mean of the inter-arrival times finite while their variance is infinite. The infinite variance syndrome is equivalent to the LRD property exhibited by the traffic as already known in literature. Moreover, the degree of long range dependency or self-similarity of the traffic is measured by the Hurst parameter denoted by H where $0 < H < 1$. It is proved that when $0.5 < H < 1$, the process has a non-summable autocorrelation function (ACF), i.e. the process exhibits the same statistical properties for different lag times or equivalently self-similar. On the other hand, if $0 < H < 0.5$, the process has a summable ACF and is said to be SRD.

The Hurst parameter H of a Pareto distributed stochastic process is related to the parameter a of the Pareto pdf as follows:

$$H = \frac{3-a}{2} \qquad (3)$$

Thus, we are going to adjust the degree of self-similarity of the generated traffic stream by the varying the parameter a between 1 and 2. More specifically, $a = 1$ corresponds to $H = 1$ which means that the generated traffic has the maximum degree of self-similarity, whereas $a = 2$ corresponds to the least degree of self-similarity when $H = 0.5$. Furthermore, as given by equation (4), the mean value of the inter-arrival time (T_a) of the generated traffic is going to be:

$$E(T_a) = \frac{1}{\lambda} = \frac{b}{a-1} \qquad (4)$$

B. Model Equations

- Case 1 (FWC case)

For the FWC case, we are going to model each output port of the OBS core node as a semi-Markov $GI/M/w/w$ queueing system which has a general independent arrivals (Pareto in our case) with a mean arrival rate λ, Markovian service times (exponential) with a mean $1/\mu$, w servers imitating the w available wavelengths and a system capacity restriction w as the OBS core node is assumed to have no buffering capabilities.

From Takacs [5], the steady-state probabilities of the *GI*/*M*/*w*/*w* queue are given in [6] and [7] as follows:

$$\pi_j = \sum_{k=j}^{w} (-1)^{k-j} \binom{k}{j} B_k \qquad \text{for } j = 0,1,2,\ldots,w \tag{5}$$

where

$$B_i = C_i \frac{\sum_{k=i}^{w} \binom{w}{k} \frac{1}{C_k}}{\sum_{k=0}^{w} \binom{w}{k} \frac{1}{C_k}} \tag{6}$$

and

$$C_r = \prod_{i=1}^{r} \frac{F^*(i\mu)}{1 - F^*(i\mu)} \tag{7}$$

where $F^*(s)$ is the Laplace transform of the pdf of the inter-arrival times. For the Pareto pdf $f_X(x)$ given by equation (1), the Laplace transform $F^*(s)$ is derived in [8] as follows:

$$F^*(s) = a(bs)^a e^{bs} \Gamma(-a, bs) \tag{8}$$

where $\Gamma(a,x)$ is the incomplete gamma function defined by:

$$\Gamma(a,x) = \int_{x}^{\infty} t^{a-1} e^{-t} dt \tag{9}$$

Furthermore, we use the same expressions in [8] in order to calculate the incomplete gamma function on MATLAB for our numerical results. The following expression is used to evaluate $F^*(s)$ if $a = n \geq 1$ is a positive integer:

$$F^*(s) = n(bs)^n e^{bs} \left[\frac{(-1)^{n-1}}{n!} \left\{ \mathrm{Ei}(-bs) - \frac{1}{2}\left(\log(-bs) - \log\left(-\frac{1}{bs}\right) \right) + \log(bs) \right\} - e^{-bs} \sum_{k=1}^{n} \frac{(bs)^{k-n-1}}{(-n)_k} \right] \tag{10}$$

where $\mathrm{Ei}(\cdot)$ denote the exponential integral function defined by:

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt \tag{11}$$

and $(x)_y = x(x+1)\ldots(x+y-1)$ is the Pochhammer symbol defined in [9].

On the other hand, if $a = n - 1/2$ is a positive half-integer then, $F^*(s)$ can be evaluated as follows:

$$F^*(s) = \left(n - \frac{1}{2}\right)(bs)^{n-1/2} e^{bs} \left[\frac{(-1)^n \sqrt{\pi}}{(1/2)_n} \mathrm{erfc}\left(\sqrt{-bs}\right) - (bs)^{1/2-n} e^{-bs} \sum_{k=0}^{n-1} \frac{(bs)^k}{(1/2-n)_{k+1}} \right] \tag{12}$$

where $\mathrm{erfc}(\cdot)$ denotes the complementary error function defined by:

$$\mathrm{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt \tag{13}$$

Intuitively, we can obtain the burst loss probability P_B by obtaining the probability that all w servers are busy, which can be easily found from equation (5) as follows:

$$P_B = \pi_w \tag{14}$$

- Case 2 (NWC case)

The NWC is similar to the FWC case except for the following. We will model each output port in the OBS core node as w independent *GI*/*M*/1/1 queues each having Pareto arrivals with a mean arrival rate λ/w. Then, we can use the same set of equations used in the FWC case above in order to calculate the burst loss probability P_B in the NWC case.

III. Numerical Results

This section is devoted to present the results of our mathematical model that assumes Pareto LRD DB arrivals to the OBS core node. Also, our model results are compared with those of conventional mathematical models that adopt Poisson SRD DB arrivals focusing on showing the effect of considering the self-similarity property possessed by traffic arrivals.

For the Poisson SRD arrivals, we employ the Erlang-B formula to derive the burst loss probability while assuming for FWC and NWC cases. For an *M*/*M*/*c*/*c* queue, the Erlang-B formula calculates the blocking probability as follows:

$$P_B = \frac{\left(\frac{\lambda}{\mu}\right)^c / c!}{\sum_{i=0}^{c} \left(\frac{\lambda}{\mu}\right)^i / i!} \tag{15}$$

First, in FWC case (γ=1), one can calculate the burst loss probability from the Erlang-B formula by simply putting the number of servers c equal to the number of wavelengths w. Second, in NWC case (γ=0), one can also use the Erlang-B formula to obtain the loss probability by putting the number of servers c equal to one while replacing the original arrival rate λ by λ/w. This is justified by the fact that each one of the w servers available is accessible only by DBs incoming on its specific wavelength which arrive by a rate λ/w, i.e. the M/M/*w*/*w* queue in case of no wavelength conversion can be replaced by w similar *M*/*M*/1/1 queues one for every wavelength.

By employing equations (5), (14) and (15), the burst loss probability P_B is plotted in Fig. 1 versus the average burst arrival rate λ in bursts/second in NWC ($\gamma = 0$) and FWC ($\gamma = 1$) cases. In both cases, we plot three curves. The first curve is for the case of Pareto distributed LRD arrivals with a Hurst parameter $H = 0.5$, i.e. the traffic exhibits a low degree of self-similarity. The second curve is for the case of Pareto distributed LRD arrivals with a Hurst parameter $H = 0.75$, i.e. the traffic exhibits a higher degree of self-similarity. The third and last curve is for the case of Poisson SRD traffic arrivals, i.e. the traffic does not exhibit any kind of self-similarity, which is drawn from the Erlang-B formula in (15).

Comparing the three curves for the NWC case, we can easily notice that P_B increases as λ increases for all curves because as the traffic arrival rate increases, the probability for contention to occur increases, and hence the probability to drop a DB increases. What is more interesting is that P_B is at its lowest value for the Poisson SRD arrivals, while P_B increases as H increases, i.e. as the degree of self-similarity increases. This is the most important observation from our model results as one can easily conclude that mathematical models depending on conventional traffic models that assume Poisson arrivals give over-optimistic results for the loss probabilities when compared to their actual values when the self-similarity property is considered. Moreover, the difference between the values of PB for SRD and LRD traffics is much clearer for lower values of λ, i.e. for low traffic scenarios. This is because heavy tailed distributions like the Pareto distribution assigns a high probability for the large values of inter-arrival times compared to the Poisson distribution. The significance of these high values of inter-arrival times is much more evident in case of light traffic values; hence P_B for LRD traffic is significantly higher than for SRD traffic for light traffic scenarios. The same observation can be easily noticed for the other three curves drawn for the FWC case. Finally, the importance of the wavelength conversion is also clear for both SRD and LRD traffics which is easily deduced from the lower values of PB for the FWC cases compared to the NWC case.

Next in Fig. 2, we plot P_B versus λ in FWC case while considering Poisson and Pareto distributed traffic arrivals with H = 0.5 and 0.75. In this figure, we want to study the effect of changing the average burst length $1/\mu$. In order to do so, we plot P_B versus λ for the three traffic cases stated for two different values of $1/\mu$. It is evident from the figure that P_B increases for larger burst length because the probability for two DBs to overlap is larger if their lengths are larger. Also, comparing the set of curves drawn at $1/\mu = 10$ with those drawn at $1/\mu = 100$, one can easily notice that as the burst length decreases, the effect of self-similarity is clearer, i.e. LRD traffic has much higher loss probabilities than SRD traffic especially when the burst length is small. The reason is simply clear; for a fixed arrival rate λ, as the burst length $1/\mu$ decreases, the traffic or the offered load ρ is smaller, i.e. a lighter traffic scenario is considered which turns the difference between SRD and LRD traffics clearer for the same reason stated previously.

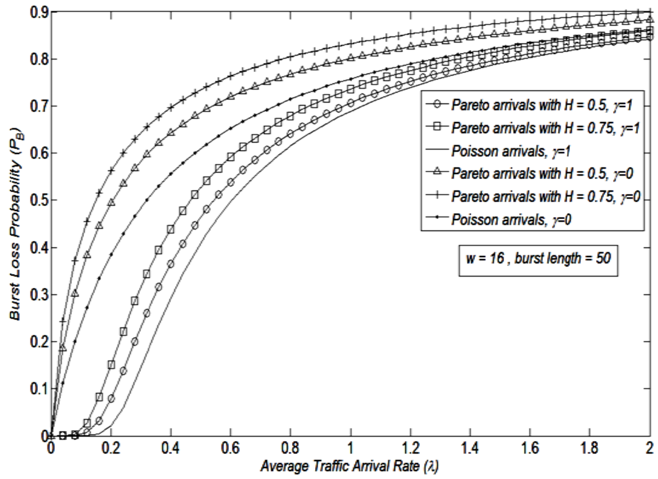

Fig. 1. Burst loss probability versus the average traffic arrival rate for NWC and FWC cases for Poisson and Pareto arrivals at different values for the Hurst parameter.

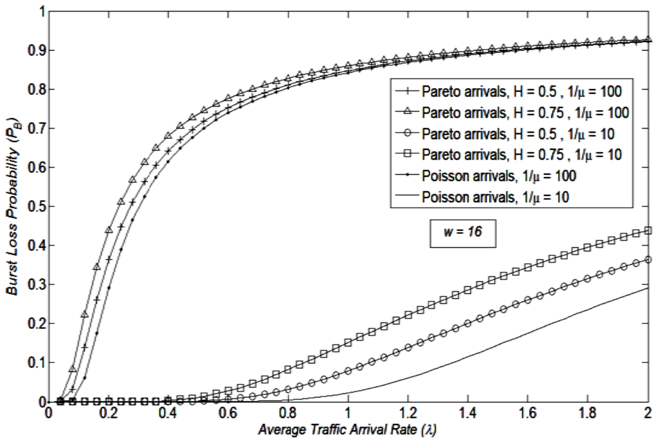

Fig. 2. Burst loss probability versus the average traffic arrival rate for FWC case for Poisson and Pareto arrivals at different values for the Hurst parameter and burst length.

Finally, we plot P_B versus the number of wavelength channels w in Fig. 3.8 for the same three traffic cases. It is intuitive that as w goes larger, P_B decreases because it is less probable for contention to occur. Interestingly, as w increases the difference between P_B values in case of SRD and LRD traffics increases, i.e. the self-similarity property is much more evident. This is because as the number of wavelengths increases the traffic per wavelength is lighter, which means that the inter-arrival times between DBs are going to be larger which is a more probable case for LRD traffic modeled by Pareto distribution rather than SRD traffic modeled by Poisson distribution.

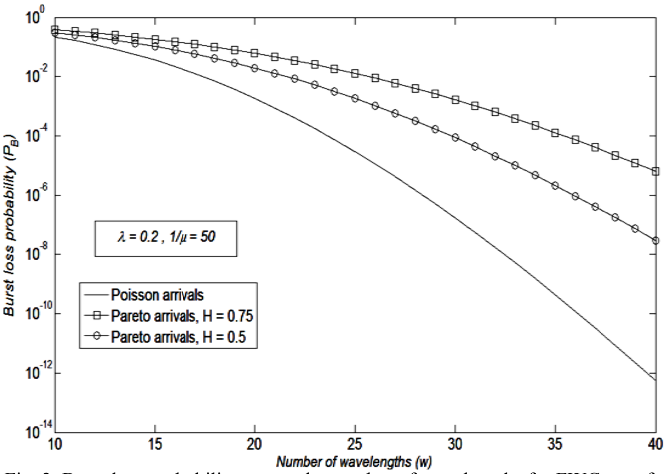

Fig. 3. Burst loss probability versus the number of wavelengths for FWC case for Poisson and Pareto arrivals at different values for the Hurst parameter.

IV. Conclusion

Upon the aforementioned results and observations, we can come up with the following conclusions:

- Results of the proposed model prove the effectiveness of adding wavelength conversion to the resources of the OBS core node for both SRD and LRD traffics.
- In spite of the disability of our proposed mathematical model to calculate the burst loss probability in PWC case while assuming self-similar or LRD traffic arrivals, the model proves the fact that other mathematical models that assume SRD Poisson traffic arrivals, thus neglecting the self-similarity property exhibited by real network traffic, are insufficient. This is because these traditional models give lower values for burst loss probabilities when compared to more complicated traffic models that consider the long range dependency, i.e. traditional models gives over-optimistic results for the loss probabilities.
- The difference between the values of the burst loss probabilities obtained from this model (Pareto LRD arrivals) and the results of the traditional Erlang-B formula (Poisson SRD arrivals) is much clearer for low traffic scenarios. Thus, for low traffic values, we should try to accurately model the real traffic LRD properties by using one of the heavy tailed distributions (Pareto for example) instead of using the conventional Poisson based models.

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