# EQUAL WEIGHT CODE SEQUENCE PER SLOT FOR OPTICAL PPM-CDMA COMMUNICATION SYSTEMS

*Hossam M. H. Shalaby, Mohab A. Mangoud*
Alexandria University, Arab Academy for Science and Technology

*Abstract* — The performance of direct detection optical fiber *pulse position modulation* (PPM) *code division multiple access* (CDMA) communication systems with *equal weight code sequence per slot* (EWPS) is investigated. Upper bounds of bit error rates for such multibit transmission with multiple access schemes are estimated considering *multiple-user interference* (MUI) in the analysis. We will investigate the performance with the assumption of Poisson shot noise model for the PIN photodiode and synchronization between users' symbols. It will be shown that the EWPS-PPM scheme outperforms the former on off orthogonal (OOO) - PPM system specially when the pulse position multiplicity is low with average power constraint on the laser be considered.

## I. INTRODUCTION

It is hoped that spread spectrum multiplexing techniques can be applied to utilize the vast bandwidth offered by the modern optical fiber technologies. We would like to investigate this possibility from system performance point of view. In previous works, optical CDMA systems concentrated on binary transmission of data, on-off keying (OOK), but in the multibit transmission Shalaby in [1] investigated the performance of multibit transmission using M-ary synchronous PPM-OCDMA communication systems under the assumption of shot noise model for the receiver photodetector. He derived a union upper bound on the BER to simplify the calculations and extracted that the performance of the system improves as M increases. Furthermore, the average power and total energy are saved by a factor log2/logM times that required in binary PPM-OCDMA. Another suggestion by Kwon is given in [2]. He proposed a system that can transmit $\log_2 L$ bits per sequence-period where L is the length of a sequence. This multibit per sequence-period system can be more power-efficient for very low bit error probability and a maximum search (instead of a threshold) in the final decision is used. Our work is an extension to the work of multibit transmission OCDMA systems. On the other hand it is shown in [3] that for OOK systems, equal weight orthogonal signaling formats that do not required dynamic estimation of the receiver threshold are preferable to the OOO signaling schemes. However employing EWO signaling format to PPM systems has not been analyzed before. We will investigate analytically the performance of direct detection M-ary PPM-OCDMA communication system under the equal weight code sequence/slot scheme with the assumption of Poisson shot noise model for the PIN photodiode and synchronization between user's symbols. The rest of the paper is organized as follows: A system model for OCDMA with EWPS-PPM signaling is described in **section II** along with an evaluation to the photon count random variables conditional expectations. **Section III** is devoted for the derivations of the bit error rate for the system. In **section IV** the effect of an optical hard limiter on the system performance is studied. Numerical and simulation results and comparisons between the performance of OOO and EWPS-PPM signaling formats are provided in **section V**. Finally the conclusion is given in **section VI**.

## II. Equal Weight Code Sequence per Slot PPM-CDMA System Description

The model of the transmitter and the receiver for this optical system is shown in Fig. 1 and 2 for only one user. This system supports N simultaneous users which transmit data continuously and synchronously. Each user is assigned two mutually orthogonal {0,1}-valued periodic spreading waveforms, $c_n(t)$ and $\overline{c}_n(t)$ of period $T_S$ and weights w and $\overline{w}$ respectively such that $w = \overline{w}$ (*equal weight code per slot* EWPS ).

Each user transmits M-ary continuous data symbols which are PPM encoded such that the T second modulation symbol interval is divided into M slots and the message of $\log_2 M$ bits is represented by the position of a pulse within one of the M possible slots, For the PPM signal of slot duration $T_S$, the *n* th user laser is modulated by $c_n(t)$ to transmit "1". A "0" is transmitted by employing

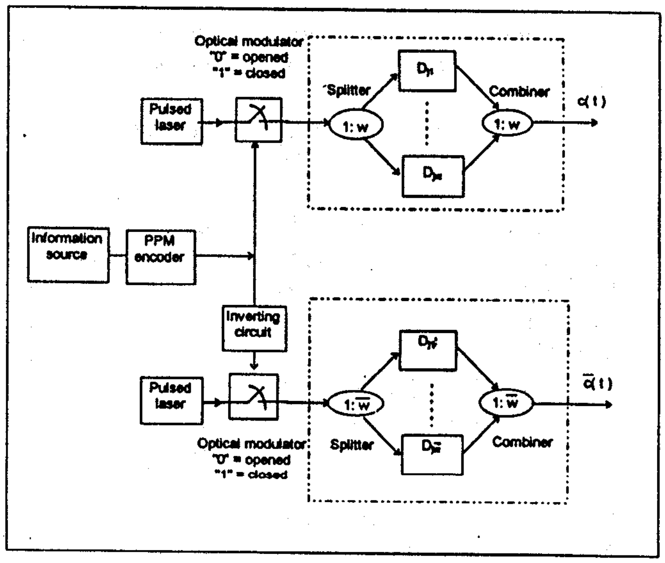


Fig.1 Optical EWPS-PPM CDMA Encoder

Hossam M. H. Shalaby is with the Department of Electrical Engineering, Faculty of Engineering, University of Alexandria, Alexandria 211544, Egypt. E-mail: shalaby@alex.eun.eg, Fax: (03) 597-1853

Mohab A. Mangoud is with the Department of Electronics and Computer, College of Engineering Studies and Technology, AAST, P.O.B. 1029 Alexandria, Egypt. E-mail: mohab@alex.eun.eg

0-7803-3567-8/96/$5.00 © 1996 IEEE

$\overline{c}_n(t)$ instead. Assume $b_n(t)$ is the nth user's (0,1) binary output of the PPM encoder which can be written as

$$b_n(t) = \sum_{j=-\infty}^{\infty} \Pi_{T_s}(t - b_{nj}T_s - jT)$$

where the PPM time frame $T=MT_s$, $T_s$ and M defined previously, $b_{nj}$ is the *j th* data sequence generated from the *n*th information sourse,$b_{nj} \in\{0,1,..,M-1\}$ and $\Pi_{T_s}(\bullet)$ is a rectangular pulse of duration $T_s$ such that :

$$\Pi_{T_s}(t) = \begin{cases} 1, & \text{if } 0 < t < T_s \\ 0, & \text{otherwise} \end{cases}$$

The two spreading signature waveforms $c_n(t)$ and $\overline{c}_n(t)$ can be written as:

$$c_n(t) = \sum_{i=-\infty}^{\infty} a_{ni}\Pi_{T_c}(t - iT_c) ,$$

$$\overline{c}_n(t) = \sum_{i=-\infty}^{\infty} \overline{a}_{ni}\Pi_{T_c}(t - iT_c)$$

where $\{a_{ni}\}$ and $\{\overline{a}_{ni}\}$ are sequences of binary optical pulses with signature length L, $a_{n(i+L)} = a_{ni}$ and $\overline{a}_{n(i+L)} = \overline{a}_{ni}$ for all integers i ,$T_c = T_s/L$ is the chip time, $\Pi_{T_c}(t)$ was defined previously and $n\in\{1,2,..,N\}$. Thus the base band signal for each user is:

$$x_n(t) = Pb_n(t)c_n(t) + P(1 - b_n(t))\overline{c}_n(t)$$

where P is the user's received laser power. The total signal waveform, (with $\Delta_n$ represents the associated delay for a given receiver) is given by

$$s(t) = \sum_{n=1}^{N} x_n(t - \Delta_n) .$$

At the receiving end, the received waveform at the front end of each receiver is

$$r(t) = s(t) + n(t)$$

Where n(t) is the optical background noise. The received optical signal is processed by two optical correlators one of them matched to one period of $c_n(t)$ and the other to one period of $\overline{c}_n(t)$ and then detected by two photodetectors. The two outputs are then filtered, sampled, subtracted from each other, and passed to the PPM decoder to choose the maximum count/slot.

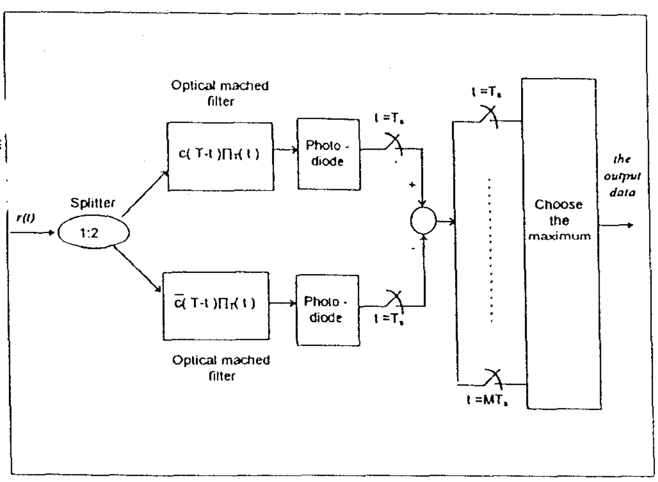


Fig.2 EWPS - PPM all optical receiver with matched filter.

In the case of synchronous schemes we can use a shifted version of the code $c_n(t)$ to be a secondary user code $\overline{c}_n(t)$ when using OOC's or any other codes with the same correlation properties. By this way the synchronous accessing schemes can accommodate a larger number of subscribers or more simultaneous users unlike asynchronous schemes in which we can not use one code and its shifted version, subsequently it accommodates less number of simultaneous users.

The received signal r(t) at the front end of each receiver is splitted into two branches. The input to the photodetector of user 1 on the first branch is given by:

$$y(t) = r(t)\, c_1(t - \Delta_1)$$

$$y(t) = Pb_1(t)[c_1(t - \Delta_1)]^2 + n(t)c_1(t - \Delta_1)$$
$$+ \sum_{n=2}^{N} Pb_n(t)c_n(t - \Delta_n)c_1(t - \Delta_1)$$
$$+ \sum_{n=1}^{N} P(1 - b_n(t))\overline{c}_n(t - \Delta_n)c_1(t - \Delta_1)$$

on the other branch, the input to the PIN diode is given by

$$\overline{y}(t) = r(t)\,\overline{c}_1(t - \Delta_1)$$
$$= P(1 - b_1(t))[\overline{c}_1(t - \Delta_1)]^2 + n(t)\overline{c}_1(t - \Delta_1)$$
$$+ \sum_{n=2}^{N} P(1 - b_n(t))\overline{c}_n(t - \Delta_n)\overline{c}_1(t - \Delta_1)$$
$$+ \sum_{n=1}^{N} Pb_n(t)c_n(t - \Delta_n)\overline{c}_1(t - \Delta_1)$$

The number of photons released from the photodetector in a $T_c$ second interval is a Poisson random variable so *the photon count over the ith slot* of the *n*th user on the two branches can be modeled as conditional Poisson random variables $Y_i^n$ and $\overline{Y}_i^n$ thus for the user 1 :

$$Y_i = Z_i + W_i + I_i \quad ,$$
$$\overline{Y}_i = \overline{Z}_i + \overline{W}_i + \overline{I}_i$$

Where $W_i$ is a Poisson photon count due to the background noise. $Z_i$ , $I_i$ are conditionally independent Poisson photon count over the *i th* slot given $\{b_{nj}\}$ and $\{\Delta_1\}$if the *i th* is the pulse containing slot (PCS). $\overline{Z}_i, \overline{I}_i$ are the same as $Z_i$, $I_i$ but if the ith slot not a PCS

Let the final decision be

$$U_i = Y_i - \overline{Y}_i$$

Then we need to evaluate the six conditional expectations for each of the above random variables, assume that the photon absorption rate due to a mark (one) transmission is $\lambda_s$ which is given by $\lambda_s = \eta P/hf$ ,where $\eta$ is the potodetector efficiency, h is Plank's constant, f is the optical frequency. We assume that the detection system is synchronous with the first user and all delays are relative to the first user delay (where $\Delta_1 = 0$). The Poisson photon count random variable $Z_i$ depends only on $b_{10}$ since $\Delta_1 = 0$. Hence its conditional expectation is

$$E[Z_i|b_{10}] = \int_{iT_s}^{(i+1)T_s} \lambda_s [c_1(t)]^2 b_1(t)\, dt$$

$$= \begin{cases} w\lambda_s T_c & \text{if } b_{10} = i \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

$$E[\bar{Z}_i|b_{10}] = \int_{iT_s}^{(i+1)T_s} \lambda_s[\bar{c}_1(t)]^2(1-b_1(t))\,dt$$

$$= \begin{cases} 0 & \text{if } b_{10} = i \\ w\lambda_s T_c & \text{otherwise} \end{cases} \quad (2)$$

Assuming the background noise photon rate is $\lambda_o$. $E[W_i] = w\lambda_o T_c$, $E[\overline{W}_i] = \overline{w}\lambda_o T_c$ (3,4)

The random variables $I_i$ and $\bar{I}_i$ are Poisson photon counts that depends only on the symbols $\{b_{n0}\}^N$ due to MUI, and their conditional means $E[I_i|\{b_{n0}\}_1^N]$ are given by:

$$\lambda_s \sum_{n=2}^{N} \int_{iT_s}^{(i+1)T_s} c_1(t)c_n(t)\Pi_{T_s}(t - b_{n0}T_s - jT)\,dt$$

$$+\lambda_s \sum_{n=1}^{N} \int_{iT_s}^{(i+1)T_s} c_1(t)\bar{c}_n(t)[1 - \Pi_{T_s}(t - b_{n0}T_s - jT)]\,dt$$

let: $I_i = \sum_{n=2}^{N} I_{1i}^n + \sum_{n=1}^{N} I_{2i}^n = I_{i1} + I_{i2}$, where,

$$E[I_{1i}^n|b_{n0}] = \lambda_s \int_{iT_s}^{(i+1)T_s} c_1(t)c_n(t)\Pi_{T_s}(t - b_{n0}T_s - jT)\,dt$$

$$= \begin{cases} \nu_{1n}\lambda_s T_c & \text{if } b_{n0} = i \\ 0 & \text{otherwise} \end{cases} = \nu_{1n}\lambda_s T_c \delta_{b_{n0},i}$$

$$E[I_{2i}^n|b_{n0}] = \lambda_s \int_{iT_s}^{(i+1)T_s} c_1(t)\bar{c}_n(t)[1 - \Pi_{T_s}(t - b_{n0}T_s - jT)]\,dt$$

$$= \begin{cases} 0 & \text{if } b_{n0} = i \\ \bar{\nu}_{1\bar{n}}\lambda_s T_c & \text{otherwise} \end{cases} = \bar{\nu}_{1\bar{n}}\lambda_s T_c(1 - \delta_{b_{n0},i})$$

where $\delta_{b_{n0},i}$ is the kronecker delta and $\nu_{1n}$, $\nu_{1\bar{n}}$ are the cross-correlation between the first and the two $n$th user codes. In our analysis we employ the optical orthogonal codes (OOC's) so we assume for simplicity $\nu_{1n} = \nu_{1\bar{n}} = 1$ for every $n \in \{1,\ldots,N\}$ hence

$$E[I_{i1}|\{b_{n0}\}_2^N] = \lambda_s T_c \sum_{n=2}^{N} \delta_{b_{n0},i}$$

$$E[I_{i2}|\{b_{n0}\}_1^N] = \lambda_s T_c \sum_{n=1}^{N} (1 - \delta_{b_{n0},i}),$$

$$E[I_i|\{b_{n0}\}_1^N] = \lambda_s T_c \sum_{n=2}^{N} \delta_{b_{n0},i} + \lambda_s T_c \sum_{n=1}^{N} (1 - \delta_{b_{n0},i})$$

$$= \lambda_s T_c\ (N-1) + \lambda_s T_c\ (1 - \delta_{b_{10},i})$$

$$E[I_i|b_{10}] = \begin{cases} (N-1)\lambda_s T_c & \text{if } b_{10} = i \\ N\lambda_s T_c & \text{otherwise} \end{cases} \quad (5)$$

similarly,

$$E[\bar{I}_i|b_{10}] = \begin{cases} N\lambda_s T_c & \text{if } b_{10} = i \\ (N-1)\lambda_s T_c & \text{otherwise} \end{cases} \quad (6)$$

from (1) through (6) we can get $E[Y_0|b_{10} = 0]$

$$= E[Z_0|b_{10} = 0] + E[W_0|b_{10} = 0] + E[I_0|b_{10} = 0]$$

$$= \sigma_s + \sigma_b + \frac{(N-1)\sigma_s}{w} \quad (7)$$

similarly,

$$E[Y_1|b_{10} = 0] = \sigma_b + \frac{N\sigma_s}{w} \quad (8)$$

$$E[\bar{Y}_0|b_{10} = 0] = \sigma_b + \frac{N\sigma_s}{w} \quad (9)$$

$$E[\bar{Y}_1|b_{10} = 0] = \sigma_s + \sigma_b + \frac{(N-1)\sigma_s}{w} \quad (10)$$

Where we defined $\sigma_s = E[Z_0|b_{10}=0] = w\lambda_s T_c$ and $\sigma_b = E[W_0|b_{10}=0] = w\lambda_o T_c$. Here $\sigma_s$ and $\sigma_b$ denote the ***average photon counts per slot due to signal and noise*** respectively. we can note that $E[W_0|b_{10}=0] = E[\overline{W}_0|b_{10}=0] = \sigma_b$ and $E[Z_0|b_{10}=0] = E[\overline{Z}_1|b_{10}=0] = \sigma_s$, since we use equal weight codes $w = \overline{w}$.

## III. Bit Error Rate of EWPS-PPM/CDMA Using The Union Bound

Assuming equally likely data symbols, the probability of symbol error decision can be upper bounded by employing a union bound on the error rate to simplify the calculations. Using (7,8,9,10) we can write :

$$P_E \le \Pr[\bigcup_{i=1}^{M-1}\{U_i \ge U_0\}|b_{10} = 0]$$

$$\le \sum_{i=1}^{M-1}\Pr\{U_i \ge U_0\}|b_{10} = 0$$

$$= (M-1)\Pr\{U_1 \ge U_0\}|b_{10} = 0\}$$

$$= (M-1)\Pr\{R \ge Q\}|b_{10} = 0\}$$

Where R and Q are also Poisson random variables (since they are sums of Poisson random variables) and their conditional expectations are

$$\mu_r = E[R|b_{10} = 0] = E[Y_1|b_{10} = 0] + E[\bar{Y}_0|b_{10} = 0]$$

$$= 2\sigma_b + 2\frac{N\sigma_s}{w}$$

$$\mu_q = E[Q|b_{10} = 0] = E[Y_0|b_{10} = 0] + E[\bar{Y}_1|b_{10} = 0]$$

$$= 2\sigma_s + 2\sigma_b + 2\frac{(N-1)\sigma_s}{w}$$

$$P_E \le (M-1)\Big[\sum_{k=0}^{\infty} e^{-\mu_r}\frac{\mu_r^{\ k}}{k!} \cdot \sum_{i=0}^{k} e^{-\mu_q}\frac{\mu_q^{\ i}}{i!}\Big]$$

Finally we can obtain the bit error propability $P_b$ from the relation in [4]which is $P_b = \frac{M/2}{M-1}P_E$

## V. Numerical Results

In our numerical evaluations we assume fixed rate constraint. Normalizing the slot width with each different value of M is thus mandatory, hence ($\tau$ = logM/RoM) and $\sigma_b = (\lambda_o/R_o)(w \log M / M L)$, where $\lambda_o/R_o$ denotes the average background noise photons per nat time. We can also defined the following parameters:

◊ $K_s$ is the average photon counts per symbol, $K_s = M\sigma_s$,

◊ $\mu$ is the average photon count per nat, $K_s = \mu \log M$,

◊ $\varphi$ is the average photon count per pulse, $K_s = Mw\varphi$.

First, we will consider the average power constraint which is equivalent to $K_s/T$ = constant or $K_s/\log M$ = constant, i.e., fixed energy per information nat, (fixed $\mu$). In our calculations we use optical orthogonal code [5] with signature length, L=341 and weight, w=5.

Fig 3 shows the union bound (U.U.B.) for the probability of bit error versus the average photon count per nat for our proposed system, with M=2,4,8 N=8 and $\lambda_o / R_o = 0$ (as reported in [1], changing this parameter with other values is not effective ). As seen in this figure, Our proposed system gives low bit error probabilities and becomes better as $\mu$ increases. For example with M=2 and $\mu$ = 300 photon/nat. the bit error rate is $Pe=3\times10^{-9}$ which gives high system performance. We can see that the BER increases as M increases, thus we can say that the best choice for pulse position multiplicity of this system is M=2. This choice gives lower BER and more simplicity in the system realization. Also, It is seen that EWPS-PPM signaling is better than OOO-PPM signaling specially at high values of $\mu$. The improvement in the performance is significant for M=2 and it is less in the case of M=4, 8, 16, because the MUI is fixed in the case of EWPS and the error will increase as M increases. It is also seen that under a constraint of the average power (or energy per information nat.) the BER decreases as M increases in the case of OOO this is due to the increase of the average photon count per pulse, $\varphi$ which is equal to $\mu \log M / w$, but in the case of EWPS the BER increases as M increases because here $\varphi$ is equal to $\mu \log M / Mw$ which decreases as M increases. we can thus conclude that when the lasers are power limited, EWPS signaling is better than OOO , because it utilizes M times the amount of average signal energy for each information transmission. We can note that with the above parameters using M=2 EWPS-PPM is better than using M=8 OOO-PPM over the rang $\mu > 50$. Compared at fixed average signal photon count per nat EWPS-PPM/ CDMA is preferable to OOO-PPM/ CDMA because it has a better performance but it has a disadvantage of the need of two signature sequences for each user in asynchronous schemes consequently there are limitations on the number of simultaneous users, but this problem can be solved when using synchronous schemes and optical orthogonal codes since we can assign a shifted version of the user code as its secondary signature sequence.

Fig. 4 and 5 shows the effect of some system parameters, as number of simultaneous users and the weight of the signature length, on the performance of the two systems. From Fig 4 it is seen that for fixed code (we assume OOC L=1365 and w=5) and fixed M (we assume M=2) the performance of each system alone gets worse (higher error rates)as N increases, we can also note that the improvement in the performance is larger in the case of EWPS when the number of simultaneous users is higher. Fig. 5 shows that for (N=8,M=2,L=631) the performance of the two systems improves as the weight of the code signature length gets higher and we can see that the EWPS schemes are better than OOO schemes with different values of w. Fig 6 shows the bit error probability versus N of EWPS-PPM/CDMA where $\mu$ =100 photons/nat w=5 and L=341. It is found that EWPS-PPM/CDMA systems with smaller M perform better at any value of N because $\varphi$ increases as M decreases, as example $\varphi$ equals to 5.19 for 8-ary PPM , 6.93 for 4-ary and 2-ary PPM.

Our results demonstrated that EWPS signaling performance compares favorably with that of OOO signaling under fixed energy levels specially with high values of average power because in the EWPS the effect of MUI is reduced. We suggest using equal weight per slot signaling when the pulse position multiplicity is low and there is a constraint on the average power.

## VI. Conclusions

The performance of direct detection optical fiber PPM/CDMA communication systems with Equal weight code sequence per slot (EWPS) signaling was suggested and investigated. Upper bounds on BERs for such multiple access schemes were estimated, taking into consideration the Poisson random nature of the photodetectors and multiple-user interference (MUI). A comparison between on-off orthogonal (OOO) and EWPS PPM/CDMA systems were studied resulting in the fact that EWPS-PPM signaling outperforms the OOO-PPM signaling for CDMA systems when the pulse position multiplicity is low under average power power constraints on the laser.

## REFERENCES


[1] H. M. Shalaby, "Performance analysis of optical synchronous CDMA communication systems with PPM signaling," to appear in *IEEE Trans. Commun.*.

[2] H. M. Kwon,"Optical Orthogonal Code Division Multiple-Access systems PartII: multibits/ sequence period," *IEEE Trans.Commun.*, vol.COM-42, pp.2592-2599, Aug.1994

[3] A. W. Lam and A.M.Hussain, "Performance analysis ofdirect detection optical CDMA communication systems with Avalanche photodiodes," *IEEE Trans. Commun.*, vol. COM-40, pp.810-820, Apr.1992.

[4] R. M. Gagliardi and S. Karp, Optical Communications. New York: John Wiley, 1976.

[5] F. R. Chung, J. A. Salehi, and V. K. Wei, "Optical Orthogonal Codes: Design, Analysis, and Applications," *IEEE Trans. Inform. Theory*, vol. I T-35 , pp.595-604, May 1989.

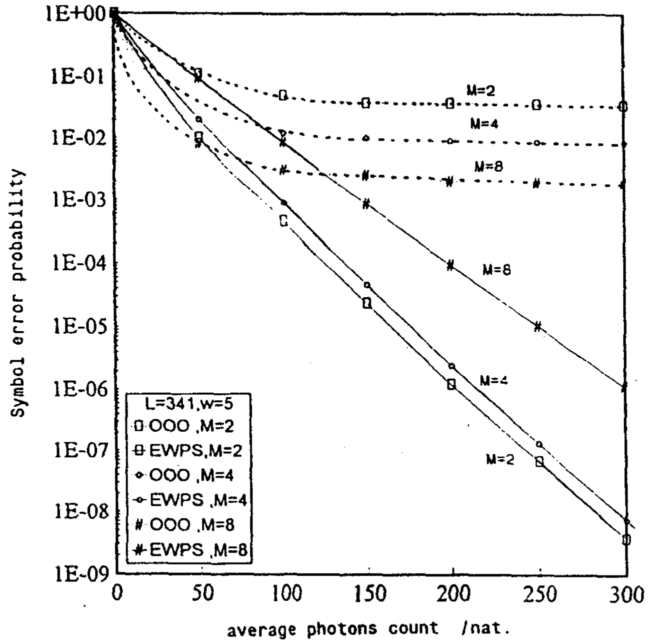


**Fig. 3** Symbol error probability versus average photons/nat, for both OOO and EWPS signaling with L=341, w=5, N=8.

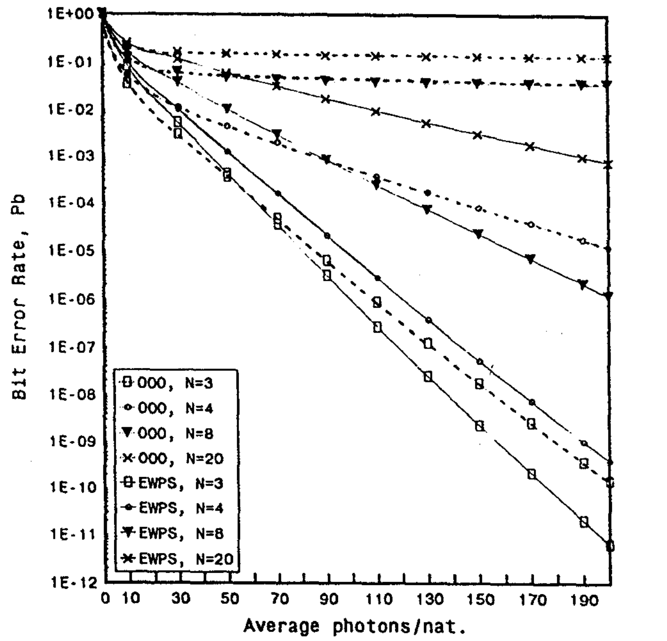


**Fig. 4** Bit error probability versus average photons /nat, for both OOO and EWPS signaling with L=1365, w=5, M=2.

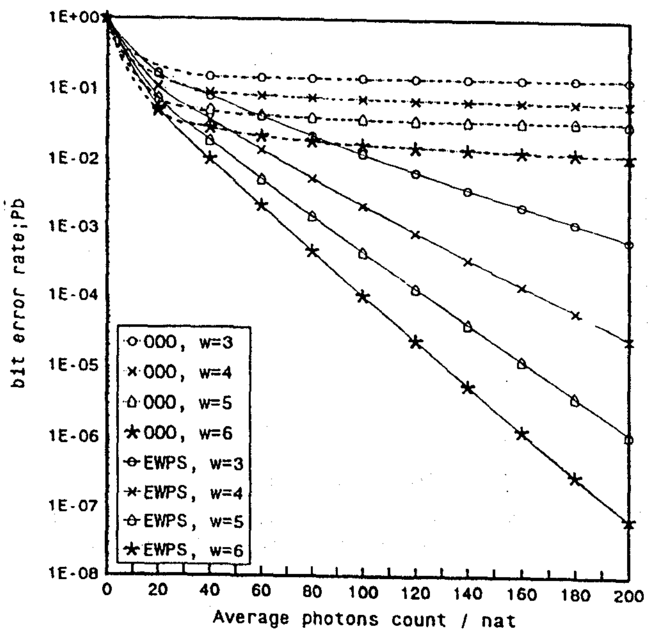


**Fig. 5** Bit error probability versus average photons/nat for EWPS and OOO signaling with L=631, N=8, M=2.

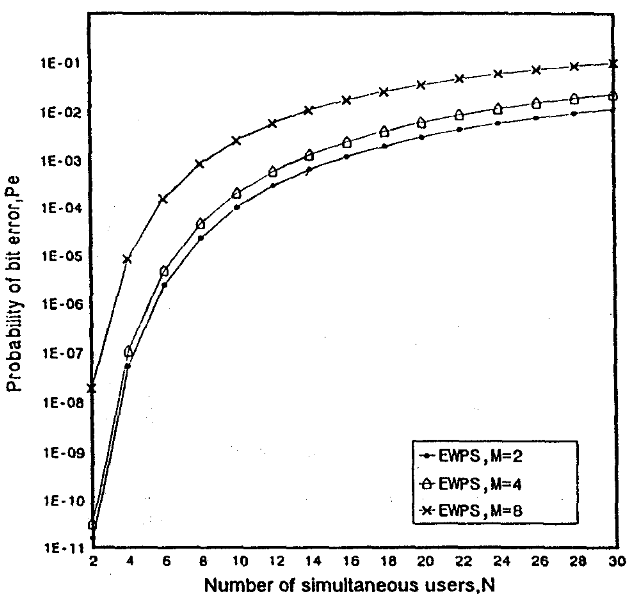


**Fig. 6** Bit error probability versus number of simultaneous users with fixed energy of 100 photons/nat.