The Capacities of Optical CDMA Communication Channels with Different Code-Correlation Constraints

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Abstract—**A comparison between the performance of several optical code-division multiple-access (CDMA) correlation receivers is presented. The performance is measured in terms of an uncoded throughput capacity. It is defined as the maximum data rate (in nats/chip time) that can be achieved with arbitrary small error probability. Both on-off keying (OOK) and pulse-position modulation (PPM) CDMA schemes are considered. Signature code correlations bounded by either one or two are employed. Our results reveal that the throughput capacity of the optical PPM-CDMA systems can be increased by increasing the code-correlation constraint from one to two. That of OOK-CDMA systems, however, cannot be increased. Further, the throughput capacity of PPM-CDMA systems with code-correlation constraint of two is greater than that of OOK-CDMA systems with code-correlation constraint of one or two. In fact, this improvement in the throughput of PPM-CDMA systems over that of OOK-CDMA approaches a limiting factor of 10 as the pulse-position multiplicity increases to infinity.**

Index Terms—**Optical SS communications, optical CDMA, code division multiple access, optical networks, channel capacity.**

I. Introduction

Optical code-division multiple-access (CDMA) techniques can be utilized in fiber-optic local area networks because of the great advantages resulting from employing high-bandwidth optical components [1]–[6]. They suffer, however, from the multiple-user interference, which degrades both the bit error probabilities and the data bit rates (of the corresponding systems) as the number of users increases. Further, they exhibit error probability floors, which cannot be reduced without the addition of interference cancellation subsystems [5]. The traditional method to recover the data at the receiving end of an optical CDMA system is to use an optical correlator followed by a photodetector and a decision device [2].

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Our goal in this paper is to evaluate the uncoded throughput capacities of both optical OOK- and PPM-CDMA correlation systems when using two different code-correlation constraints, namely $\lambda \in \{1, 2\}$. The uncoded throughput capacity is defined as the maximum data rate (in nats/chip time) that can be transmitted with arbitrary small error probability. We employ the optical orthogonal codes (OOCs) [1], with periodic cross-correlations and out-of-phase periodic auto-correlations that are bounded by either one or two ($\lambda \in \{1, 2\}$), as the users' signature code sequences in our theoretical analysis.

The remainder of this paper is organized as follows. The optical OOK- and PPM-CDMA receiver models are described in Section II. The uncoded throughput capacity of the optical OOK-CDMA system, under code correlations bounded by one, is derived in Section III. Section IV is devoted for the development of the uncoded throughput capacity of the optical OOK-CDMA system under code-correlation constraint equal to two. In Section V, we derive the corresponding results for PPM-CDMA systems. In Section VI, we compare between the uncoded throughput capacities of both OOK- and PPM-CDMA systems with different code-correlation constraints. Finally the conclusion is given in Section VII.

II. Optical OOK- and PPM-CDMA Receiver Models

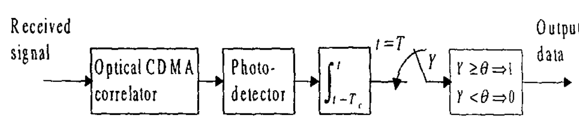

Fig. 1. An optical OOK-CDMA correlation receiver.

The block diagram for the optical OOK-CDMA correlation receiver is shown in Fig. 1, where T and T_c denote the bit time and the chip time durations, respectively. The optical CDMA correlator usually splits the received optical signal into a number of branches,

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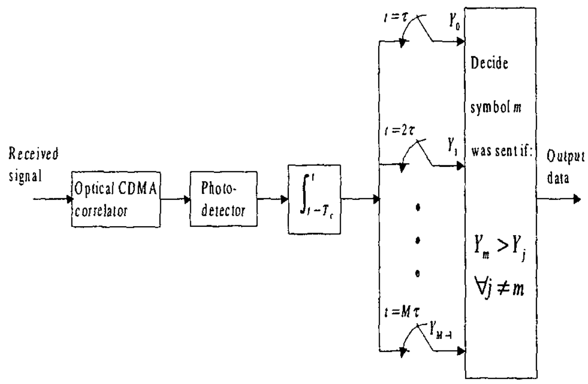

Fig. 2. An optical PPM-CDMA correlation receiver.

which is equal to the code weight w, and then combines these branches after properly delaying the split optical pulses in accordance to the signature code. The electronic switch in Fig. 1 samples at a rate that is equal to the data bit rate $R_b = 1/T$. This rate is much less than the optical processing rate $R_c = 1/T_c$. In fact $R_b = R_c/L$, where L is the code length.

The corresponding block diagram for the PPM-CDMA correlation receiver is shown in Fig. 2, where M denotes the pulse-position multiplicity and τ denotes the slot time duration. The transmitted data takes values in the set $\{0, 1, \ldots, M-1\}$. The electronic switches sample at the end of all M slots. The index of the maximum sample is declared to be the transmited data.

III. Uncoded Throughput Capacity of OOK-CDMA Systems with $\lambda = 1$

The bit error probability for the correlation receiver of Fig. 1, with $\lambda = 1$, is given by

$$P_b = \frac{1}{2}\left(P[E|0] + P[E|1]\right) , \tag{1}$$

where, for a given threshold θ,

$$P[E|0] = \Pr\{\kappa_1 \geq \theta\}$$

and

$$P[E|1] = Pr\{\kappa_1 < \theta - w\} .$$

Here κ_1 denotes the number of users that cause interference to the desired user at one pulse position. It can be modeled as a binomial random variable:

$$\Pr\{\kappa_1 = l\} = \binom{N-1}{l} p_1^l (1-p_1)^{N-1-l} ,$$

where N denotes the number of simultaneous users, $l \in \{0, 1, \ldots, N-1\}$, and p_1 denotes the probability that a single user interferes with the desired user at only one pulse position. It is given by

$$p_1 = \frac{w^2}{2L} .$$

If we choose an optimum threshold [2], $\theta = w$, then

$$\begin{aligned} P_b &= \frac{1}{2} Pr\{\kappa_1 \geq w\} \\ &= \frac{1}{2} \sum_{l=w}^{N-1} \binom{N-1}{l} p_1^l (1-p_1)^{N-1-l} . \end{aligned}$$

Using Chernof bound we can find an upper bound to the last probability: For any $z \geq 1$

$$\begin{aligned} P_b &\leq z^{-w} \frac{1}{2} \sum_{l=0}^{N-1} \binom{N-1}{l} z^l p_1^l (1-p_1)^{N-1-l} \\ &= \frac{1}{2} z^{-w} \left(1 - p_1 + z p_1\right)^{N-1} \\ &\leq \frac{1}{2} z^{-w} \exp[N z p_1] . \end{aligned}$$

By finding $z \geq 1$ for the tightest bound, we get

$$z = \frac{w}{N p_1} = \frac{2L}{wN} \geq 2(w-1) .$$

The last inequality holds because of the condition on the available number of OOCs [2]:

$$N \leq \frac{L-1}{w(w-1)} . \tag{2}$$

Hence

$$P_b \leq \frac{1}{2}\left(\frac{N p_1 e}{w}\right)^w = \frac{1}{2}\left(\frac{N w e}{2L}\right)^w .$$

In order to satisfy the error constraint $P_b \leq \epsilon$, then

$$N \leq \frac{2L}{we} \exp\left[-\frac{\alpha - \log 2}{w}\right] , \tag{3}$$

where $\alpha = -\log \epsilon$. The uncoded throughput capacity is defined as

$$R_0 \stackrel{\text{def}}{=} \max_{\substack{N,L: \\ P_b \leq e^{-\alpha}}} N \cdot \frac{\log 2}{L} \qquad \text{nats/chip time} .$$

It can be lower bounded by choosing the achievable value of N from the right-hand side of (3).

$$R_0 \geq \frac{2 \log 2}{we \exp\left[\frac{\alpha - \log 2}{w}\right]} .$$

Further from the constraint on N given in (2), the throughput cannot increase above

$$\frac{\log 2}{w(w-1)} \cdot \frac{L-1}{L} .$$

Thus the R_0 is lower bounded by

$$R_0 \geq \min\left\{\frac{2\log 2}{we\exp\left[\frac{\alpha-\log 2}{w}\right]}, \frac{\log 2}{w(w-1)} \cdot \frac{L-1}{L}\right\} .$$

For a sufficiently large value of L, we obtain

$$R_0 \geq \min\left\{\frac{2\log 2}{w\exp\left[\frac{\alpha-\log 2}{w}+1\right]}, \frac{\log 2}{w(w-1)}\right\} . \quad (4)$$

IV. Uncoded Throughput Capacity of OOK-CDMA Systems with $\lambda = 2$

Let p_t, $t \in \{1, 2\}$, denote the probability that a single user interferes with the desired user at exactly t pulse positions. It was shown in [3] that

$$p_1 + 2p_2 = \frac{w^2}{2L} .$$

In this section, we only study the worst case, which occurs when $p_1 = 0$ in the last equation, and hence

$$p_2 = \frac{w^2}{4L} . \quad (5)$$

That is, if a single user interferes with the desired user, it will cause interference to exactly two mark positions of the desired user. The system performance in this case provides an upper bound to the more general one with $p_1 \geq 0$ [3]. Let κ_2 denotes the number of users that cause interference to the desired user at two pulse positions. Then

$$\Pr\{\kappa_2 = l\} = \binom{N-1}{l} p_2^l (1-p_2)^{N-1-l} ,$$

where $l \in \{0, 1, \ldots, N-1\}$. The optimum error probability is given by

$$P_b = \frac{1}{2}Pr\{2\kappa_2 \geq w\} \leq \frac{1}{2}Pr\{\kappa_2 \geq w/2\} .$$

Following a similar argument to that in Section III we get, for any $z \geq 1$

$$P_b \leq \frac{1}{2} z^{-w/2} \exp[Nzp_2] .$$

By finding $z \geq 1$ for the tightest bound, we get

$$z = \frac{w}{2Np_2} = \frac{2L}{wN} .$$

But the condition on the available number of OOCs when $\lambda = 2$ [3], is now given by

$$N \leq \frac{(L-1)(L-2)}{w(w-1)(w-2)} . \quad (6)$$

Hence the code length should be chosen so as $z \geq 1$. In fact if we let

$$L \leq 2(w-1)(w-2) + 2 , \quad (7)$$

then $z > 1$. In our analysis in this section we take L equal to the RHS of (7). The error rate is now given by

$$P_b \leq \frac{1}{2}\left(\frac{2Np_2e}{w}\right)^{w/2} = \frac{1}{2}\left(\frac{Nwe}{2L}\right)^{w/2} .$$

In order to satisfy the error constraint $P_b \leq \epsilon = e^{-\alpha}$, then

$$N \leq \frac{2L}{we} \exp\left[-\frac{2(\alpha - \log 2)}{w}\right] . \quad (8)$$

The corresponding throughput capacity is thus

$$R_0 \geq \frac{2\log 2}{w\exp\left[\frac{2(\alpha-\log 2)}{w}+1\right]} . \quad (9)$$

Further from the constraint on N given in (6), the throughput cannot increase above

$$\frac{\log 2}{w(w-1)(w-2)} \cdot \frac{(L-1)(L-2)}{L} ,$$

which is always greater than the RHS of (9) for a value of L given by the RHS of (7). Thus (9) gives the required bound on the throughput capacity when $\lambda = 2$.

V. Uncoded Throughput Capacity of PPM-CDMA Systems with $\lambda \in \{1, 2\}$

We denote by κ_j, $j \in \{0, 1, \ldots, M-1\}$, the number of users that interfere with the desired user in slot j. Thus for any $\lambda \in \{1, 2\}$, the word error probability can be written as

$$\begin{aligned} P_M &= \frac{1}{M}\sum_{i=0}^{M-1} \Pr\{\lambda\kappa_j \geq \lambda\kappa_i + w, \text{ some } j \neq i | \\ &\qquad\qquad i \text{ was sent}\} \\ &= \Pr\{\lambda\kappa_j \geq \lambda\kappa_0 + w, \text{ some } j \neq 0\} \\ &\leq \Pr\{\lambda\kappa_j \geq w, \text{ some } j \neq 0\} . \end{aligned}$$

Applying a union bound, we have for any $z \geq 1$

$$P_M \leq (M-1)\Pr\{\kappa_1 \geq \frac{w}{\lambda}\} = (M-1)\Pr\{z^{\kappa_1} \geq z^{w/\lambda}\}.$$

Here κ_1 is again a binomial random variable with parameters $N-1$ and p_1:

$$\Pr\{\kappa_1 = l\} = \binom{N-1}{l} p_1^l (1-p_1)^{N-1-l}$$

and

$$p_1 = \frac{w^2}{\lambda M L}.$$

Using the Markov inequality, we get

$$\begin{aligned} P_M &\leq (M-1)z^{-w/\lambda} E\{z^{\kappa_1}\} \\ &= (M-1)z^{-w/\lambda}(1-p_1+zp_1)^{N-1}. \end{aligned}$$

The corresponding bit error rate is given by

$$\begin{aligned} P_b = \frac{M}{2(M-1)} P_M &\leq \frac{M}{2} z^{-w/\lambda}(1-p_1+zp_1)^{N-1} \\ &\leq \frac{M}{2} z^{-w/\lambda} \exp[Nzp_1]. \end{aligned} \quad (10)$$

The best $z \geq 1$ is given by

$$z = \frac{w}{\lambda N p_1} = \frac{ML}{wN}.$$

From (2) and (6) L should be restricted as follows to satisfy the condition $z > 1$:

$$L \leq \begin{cases} \infty\,; & \text{if } \lambda = 1, \\ M(w-1)(w-2)+2\,; & \text{if } \lambda = 2. \end{cases} \quad (11)$$

By substituting the last value of z into (10), we get

$$P_b \leq \frac{M}{2}\left(\frac{Nwe}{ML}\right)^{w/\lambda}.$$

Invoking the error constraint $P_b \leq \epsilon = e^{-\alpha}$, we obtain

$$N \leq \frac{ML}{we} \exp\left[-\frac{\lambda(\alpha + \log M - \log 2)}{w}\right]. \quad (12)$$

The uncoded throughput capacity for PPM-CDMA systems is defined as

$$R_0 \stackrel{\text{def}}{=} \max_{\substack{N,L: \\ P_b \leq e^{-\alpha}}} N \cdot \frac{\log M}{ML}.$$

From (12), it can be lower bounded as

$$R_0 \geq \frac{\log M}{w \exp\left[\frac{\lambda\{\alpha + \log(M/2)\}}{w} + 1\right]}. \quad (13)$$

Further from the constraints on N given in (2) and (6), the throughput cannot increase above

$$\begin{aligned} \frac{\log M}{Mw(w-1)} \cdot \frac{(L-1)}{L}\,; &\quad \text{if } \lambda = 1, \\ \frac{\log M}{Mw(w-1)(w-2)} \cdot \frac{(L-1)(L-2)}{L}\,; &\quad \text{if } \lambda = 2. \end{aligned}$$

But for $\lambda = 2$, if we choose $L = M(w-1)(w-2) + 2$, then the second constraint is always greater than the RHS of (13). Thus the final lower bound on the throughput capacity can be written as.

$$R_0 \geq \begin{cases} \min\left\{\frac{\log M}{w \exp\left[\frac{\alpha + \log(M/2)}{w} + 1\right]}, \frac{\log M}{Mw(w-1)}\right\}; & \lambda = 1, \\ \frac{\log M}{w \exp\left[\frac{2\{\alpha + \log(M/2)\}}{w} + 1\right]}; & \lambda = 2. \end{cases} \quad (14)$$

VI. Numerical Results

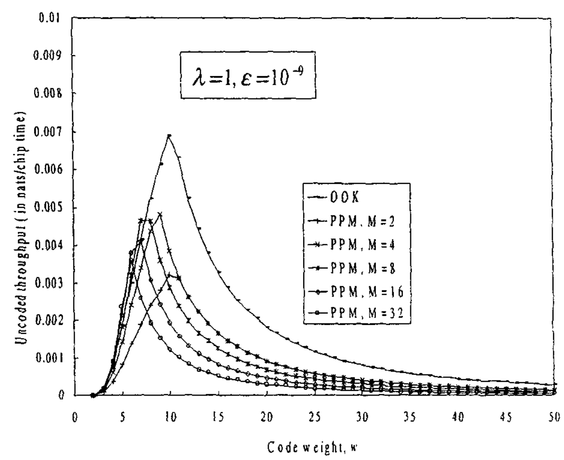

Fig. 3. Uncoded throughput capacities versus the code weight for both OOK- and PPM-CDMA correlation systems with λ=1.

The throughput capacities of both OOK- and PPM-CDMA correlation systems with $\lambda = 1$ are shown in Fig. 3 versus the code weight for different values of M and an error rate constraint of $\epsilon = 10^{-9}$. It can be seen that an optimum value of the code weight always exist. Further for PPM-CDMA systems, an optimum value of M also exists. It can be seen that the OOK-CDMA systems offer a higher throughput than PPM-CDMA systems for all values of M. This maximum achievable throughput is about 0.00688 nats/chip time. The main limitation of the throughput is in fact due to the codewords constraint as given in (2).

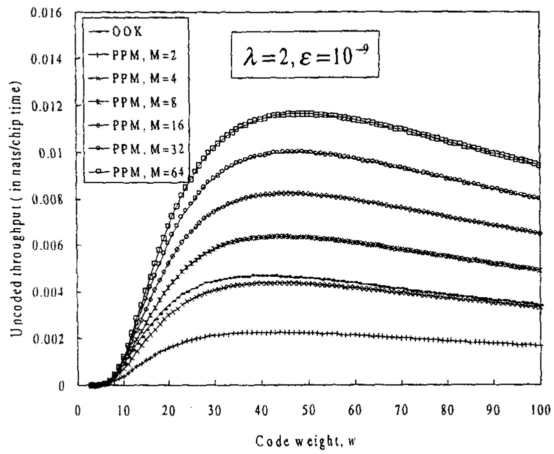

Fig. 4. Uncoded throughput capacities versus the code weight for both OOK- and PPM-CDMA correlation systems with λ=2.

The corresponding throughput capacities of both OOK- and PPM-CDMA correlation systems with $\lambda = 2$ are shown in Fig. 4 versus the code weight for different values of M and an error rate constraint of $\epsilon = 10^{-9}$. The codewords constraint in this case is now relaxed as given by (6) and thus is not a limiting factor anymore. Again it can be seen that an optimum value of the code weight always exist for both OOK- and PPM-CDMA systems. An optimum value of M does not exist, however, for PPM-CDMA systems (as was in the case of $\lambda = 1$). This is because of the relaxed limitation on the codewords constraint. It is also obvious from the figure that the PPM-CDMA systems offer a higher throughput than OOK-CDMA systems, in this case, for values of M greater than 4. For example, the maximum achievable throughput is about 0.0116 nats/chip time, when $M = 64$.

When comparing Fig. 3 to Fig. 4, we find that there is no improvement in the throughput of OOK-CDMA systems when we increase λ from 1 to 2. This is because much interference will be added when using $\lambda = 2$. An improvement exists, however, in the throughput of PPM-CDMA systems when we increase λ from 1 to 2. This is because we can increase the pulse-position multiplicity M to compensate for the extra interference with $\lambda = 2$. The improvement in the throughput of PPM-CDMA with $\lambda = 2$ makes it even better than that of OOK-CDMA with $\lambda = 1$.

We can get the optimum code weights of the PPM-CDMA systems of Fig. 4 by differentiating the second line of (14) with respect to w and equating the result to zero. We obtain

$$w_{opt} = 2\Big(\alpha + \log\frac{M}{2}\Big) .$$

The corresponding maximum throughput is thus

$$R_{0,opt} \geq \frac{\log M}{2e^2[\alpha + \log(M/2)]} .$$

The extreme reachable throughput can be obtained by taking the limit of the last equation when $M \to \infty$.

$$R_{0,opt,limit} \geq \frac{1}{2e^2} = 0.068 .$$

We notice that this limit is almost 10 times greater than the throughput of OOK-CDMA with $\lambda = 1$.

VI. Conclusion

The uncoded throughput capacity for both optical OOK- and PPM-CDMA correlation systems are derived and evaluated under code correlations bounded by one and two. We have the following concluding remarks.

i) PPM-CDMA receivers with code correlation constraint of two ($\lambda = 2$) are very efficient when compared to OOK-CDMA receivers with $\lambda \in \{1,2\}$ and to PPM-CDMA receivers with $\lambda = 1$.
ii) An optimum value of the code weight always exist for any given constraint on the bit error rate.
iii) For PPM-CDMA systems with $\lambda = 1$, an optimum value of M also exists. For $\lambda = 2$, however, no such an M exists, and the optimum throughput increases with M till it reaches a finite limit of 0.068 nats/chip time.
iv) The maximum limiting throughput of PPM-CDMA systems with $\lambda = 2$ is about 10 times greater than that of OOK-CDMA with $\lambda \in \{1,2\}$.

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