Analysis of Frequency Reuse Cellular Systems Using Worst Case Signal to Interference Ratio

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Abstract—Frequency reuse schemes are an interference management techniques well suited to OFDMA-based cellular networks. In this paper, analytical expressions are derived for the worst case Signal to Interference Ratio (SIR) ratio for both Fractional Frequency Reuse (FFR), and Soft Frequency Reuse (SFR). Also, analytical expressions are derived for the best inner radius using the worst SIR in both FFR, and SFR schemes. The analysis is performed in cellular network using exponent path loss model. The results showed that FFR with reuse four has the smallest interference hence better edge spectral effiency than SFR with different power ratios. For SFR it is better to use power ratios between 2, 4 as they achieve reasonable inner radii than other power ratios. The analysis showed that there is tradeoff aspect between capacity and coverage related to SFR and FFR respectively.

Keywords: ICI, FFR, FRF, SFR, and optimum inner radius.

I. INTRODUCTION

LTE technology is a basic mobile communication standard presented in late 2009 by ITU-T. Nowadays 4th Generation of Mobile communication systems are launched known as LTEadvanced. The main targets of LTE system are to support high data transfer, low latency, increased bandwidth (capacity), and improve Quality Of Service (QOS). However these benefits face a lot of challenges. Among these challenges are high path loss and greater signal attenuation due to higher frequencies, transmit power controls, and the problem of interfering signals from neighbor cells. The last problem is known as Inter Cell Interference (ICI). It results from the motion of users from cell center to cell edge resulting in power reduction of the signal transmitted from the cell center while interference signals from neighbor cells is increased. ICI mitigation techniques are a hot topic research area in wireless communications.

Inter cell interference randomization, cancellation, and coordination/avoidance are three general approaches for ICI mitigation approaches [1, 3, 4, 9]. Frequency reuse is one of the most commonly used interference coordination technique, where the whole frequency band is divided into several subbands and wisely allocated to a specific area so as to improve signal status at cell edge.

Frequency reuse is a common approach to increase data rate of point to multipoint systems. There are three main types of frequency reuse schemes for ICI mitigation; they are: Hard frequency reuse, FFR, and SFR [4]. Hard frequency reuse divides the system bandwidth into a number of sub-bands according to the reuse factor. Neighboring cells transmit on different sub bands. Hard frequency reuse with reuse 3 is shown in fig.1.a. For an ideal hexagonal lattice, frequency reuse factor r equals $i^2 + ij + j^2$, where i, j are non-negative integers. For example i=1, j=0 yields reuse one where each cell reuses all the entire bandwidth at the cost of high interference. i=1, j=1 yields reuse three where each cell may use one third of the system bandwidth so as to reduce interference at the cell edges. The most common reuse factors for hexagonal lattice are 1, 3, 4, 7, 9, 12, etc.

FFR splits the bandwidth into inner and outer sub bands. The inner part lies near the base station at the center of the cell. It uses frequency reuse one. The outer part lies in the cell edges and it uses frequency reuse larger than one for ICI mitigation at the cell edges. FFR is an effective and feasible solution for ICI problem as it improves the throughput of celledge user. The idea was firstly applied in GSM networks [7], and was adopted by WIMAX system later [5, 8], and also absorbed in LTE standard [9]. FFR with Frequency Reuse Factor (FRF) = 3 is shown in fig.1.b. On the other hand for SFR case, the overall bandwidth is shared by all base stations [10]. The bandwidth is divided into major and minor sub bands with power control. Major sub band is used for both center and edge users, while minor sub band is used only for center users. The transmission power for major sub-band is larger than minor sub band power. SFR with reuse three is shown in fig.1.c.

In [12] the authors proposed an analytical model for FFR schemes for modelling base station locations using homogenous Poisson point process (PPP). The authors in [13] proposed analytical models for Integer Frequency Reuse (IFR), FFR, and Two level power control (TLPC) schemes. The proposed models are based on the fluid model proposed [14]. The models were found to be time efficient and close to hexagonal models. In [18] the authors evaluated the effect of SFR on Spectral Effiency (SE) depending on power allocation schemes and the number of sub bands. The comparison between FFR 3 and 4 is discussed in [19]. It was found that FRF=4 has better performance and in general it depends on the used topology. An Adaptive SFR for OFDMA is proposed in [20]. It improves the capacity of SFR by jointly optimizing subcarrier and power allocation. A novel FFR scheme combined with interference suppression for OFDMA networks is discussed in [21]. Interference suppression scheme saves power and achieves the required network QOS with low complexity. An interesting comparison of FFR schemes is discussed in [22]. The authors resulted that strict FFR provides the greatest overall network throughput and highest cell edge user SIR, while SFR archives the requirements of interference reduction and resource effiency.In [2] the author determined the optimal frequency reuse factor of the exterior users as well as the bandwidth to assign to both interior and exterior zones. The optimal interior radius is determined approximately 2/3 of the overall cell radius. In [11] theoretical capacity and outage rate of an OFDMA cellular system employing FFR have been analysed. Numerical results showed that FFR achieves higher capacity than non FFR when outage rate is low.

In this paper we focus on the analysis of Cell Edge Users (CEUs). An optimum inner radius based on worst case SIR in cellular system is given. The worst SIR is found as a function of the path loss exponent basically for FFR and with the power ratio in case of SFR. Paper organization is as follows: Section 2 previews the system model and derives the worst SIR and Optimum inner radius for both FRF=3and FRF=4. Section 3 repeats the same analysis for SFR. Section 4 presents simulation results. Finally section 5 concludes the work.

II. SYSTEM MODEL FOR FFR

Consider a two tier cellular system that utilizes FFR with FRF=3 and FRF=4 as shown in Figs.2, 3 respectively. The total bandwidth equals BW_{tot}=B1+FRF*B2. Where B1 is the BW allocated to reuse one region and the rest of the BW is divided equally either into three equal values as shown in Fig. 2 or four equal parts as shown in Fig.3 according to the FRF value. Each BS uses omnidirectional antenna. The distance between any two adjacent Base Stations (BS's) is 2d where $d=\frac{\sqrt{3}}{2}R$, and R is the cell radius. Assume constant and uniform user density. A total transmit power P_{tot} is available at each BS. Each BS transmits with a constant power spectral density p=P_{tot}/B_{tot}. Channel variations due to Path loss is considered, where we use the exponent path loss model [15, 16] given by $G=G_0r^{-\alpha}$, where α is the path loss exponent. It depends on the terrain nature and on antenna height. The parameter r is the distance between the base station and User Equipment (UE).

The constant G_0 is given by $G_0=(c/(4\pi f))^2$ where f is the center frequency and c is the speed of light.

If a user lies in U(X, Y) at a distance $r=\sqrt{(X^2 + Y^2)}$ from BS0 (see Figs.2, 3), the user is either inner or exterior user depending on its location. New parameter m is defined for this purpose if m=1 then the user is CCU and if m=0 then the user is CEU. For inner user we consider eighteen interfering BS on that user BS₁:BS₁₈. For exterior user we consider six nearest interfering BSs white coloured cells in Figs2, 3.



Fig.1 a) Reuse three. b) Strict FFR. c) Soft FFR.

Thus a general SIR expression at point (X, Y) is as follows:

$$SIR(X,Y) = \frac{phG_0r^{-\alpha}}{m\sum_{k=1}^{k=18} ph_kG_0 \ r_k^{-\alpha} + m'\sum_{i=1}^{i=6} ph_iG_0 \ r_i^{-\alpha}}(1)$$

where r_k is the distance between U and BS_k and r_i is the distance between exterior user and six interfering BSs. m[\] is the complement of parameter m. h is the exponentially distributed channel fading power. For simplicity of the analysis, we assume that channel fading powers are independent with unit mean. Let (x, y) be the normalized coordinates to R, i.e (x, y) = (X/R, Y/R). The normalized coordinates for interfering BS's are shown in table1. It follows from (1) that SIR expression in normalized coordinates becomes

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$$sir_{CCU}(x,y) = \frac{(x^2 + y^2)^{-\frac{\alpha}{2}}}{S1(x,y)}$$
(2)

where S1(x, y) is the interference due to reuse one frequency from the eighteen BSs. It is given by:

$$S1(x,y) = \left[\left(x \pm \sqrt{3} \right)^2 + y^2 \right]^{-\frac{\alpha}{2}} + \left[\left(x \pm 2\sqrt{3} \right)^2 + y^2 \right]^{-\frac{\alpha}{2}} + \left[\left(x \pm \frac{\sqrt{3}}{2} \right)^2 + \left(y \pm \frac{3}{2} \right)^2 \right]^{-\frac{\alpha}{2}} + \left[\left(x \pm \frac{3\sqrt{3}}{2} \right)^2 + \left(y \pm \frac{3}{2} \right)^2 \right]^{-\frac{\alpha}{2}} + \left[x^2 + \left(y \pm 3 \right)^2 \right]^{-\frac{\alpha}{2}} + \left[\left(x \pm \sqrt{3} \right)^2 + \left(y \pm 3 \right)^2 \right]^{-\frac{\alpha}{2}}$$
(3)

For CEU equation (1) will change to

$$sir_{CEU}(x, y) = \frac{(x^2 + y^2)^{-\frac{\alpha}{2}}}{SFRF(x, y)}$$
(4)

The parameter SFRF is the interference factor due to FFR. Equation (5) describes SFRF for FRF=3. SFRF for FRF=4 is described in (6).

$$SFRF3(x,y) = [(x \pm \frac{3\sqrt{3}}{2})^2 + (y \pm \frac{3}{2})^2]^{-\frac{\alpha}{2}} + [x^2 + (y \pm 3)^2]^{-\frac{\alpha}{2}}$$
(5)

$$SFRF4(x,y) = [(x \pm 2\sqrt{3})^2 + y^2]^{-\frac{\alpha}{2}} + [(x \pm \sqrt{3})^2 + (y \pm 3)^2]^{-\frac{\alpha}{2}}$$
(6)

TABLE I BSs Coordinates and Assigned Frequencies for FFR and SFR							
			FFR=3	FR=3 FFR=4		SFR	
BS	х	Y	Edge	Edge	Center	Edge	
			freq	freq	freq	freq	
0	0	0	f2(white)	f2(white)	f2,f3	f1	
1	$\sqrt{3}$	0	f3(blue)	f3(blue)	f1,f3	f2	
2	$\sqrt{3/2}$	3/2	f4(green)	f4(green)	f1,f2	f3	
3	- √ 3/2	3/2	f3	f5(pink)	f1,f3	f2	
4	-√3	0	f4	f3	f1,f2	f3	
5	-\sqrt{3/2}	-3/2	f3	f4	f1,f3	f2	
6	$\sqrt{3/2}$	-3/2	f4	f5	f1,f2	f3	
7	2√3	0	f4	f2	f1,f2	f3	
8	3√3/2	3/2	f2	f5	f2,f3	f1	
9	$\sqrt{3}$	3	f3	f2	f1,f3	f2	
10	0	3	f2	f3	f2,f3	f1	
11	-√3	3	f4	f2	f1,f2	f3	
12	-3\sqrt{3/2}	3/2	f2	f4	f2,f3	f1	
13	-2√3	0	f3	f2	f1,f3	f2	
14	-3\sqrt{3/2}	-3/2	f2	f5	f2,f3	f1	
15	-√3	-3	f4	f2	f1,f2	f3	
16	0	-3	f2	f3	f2,f3	f1	
17	$\sqrt{3}$	-3	f3	f2	f1,f3	f2	
18	3\sqrt{3/2}	-3/2	f2	f4	f2,f3	f1	

An important question that one should ask what is the worst SIR case for the cellular system shown in figs.2, and 3. The answer is the SIR at the corners of the hexagon in center cell for exterior users. For the interior user it is at the inner circle point (R_{in, 0}). The coordinates of the corners of hexagon are $(\frac{\sqrt{3}}{2},\frac{1}{2})$, (0,1), $(\frac{-\sqrt{3}}{2},\frac{1}{2})$, $(-\frac{\sqrt{3}}{2},-\frac{1}{2})$, (0,-1), and $(\frac{\sqrt{3}}{2},-\frac{1}{2})$. Consider a user located at the first corner with coordinates $(\frac{\sqrt{3}}{2},\frac{1}{2})$. It's worst SIR in case of FRF=3 and FRF=4 is given by (7) and (8) respectively.



Fig.2. Two tiers network with FRF=3



Fig.3. Two tiers network with FRF=4

worstSIR_{FRF=3} = $\frac{1}{(2^{-\alpha} + 2 * 13^{-\frac{\alpha}{2}} + 2 * 7^{-\alpha/2} + 4^{-\alpha/2})}$ (7)

worst SIR _{FRF=4} =
$$\frac{1}{2 * [13^{-\frac{\alpha}{2}} + 7^{-\frac{\alpha}{2}} + 19^{-\frac{\alpha}{2}}]}$$
 (8)

From (6, 7) the worst SIR depends basically on path loss exponent α . There are a lot of optimization techniques

proposed for the inner cell radius as in [2, 17]. In these techniques the authors converted the optimization problem from non-convex into convex problem (Geometric programming (GP)). In order to find the optimal FFR inner radius, the approach divides the home cell into two regions (inner and exterior) and calculates the worst SIR for the two cases .The worst SIR for reuse three and four are given previously in (7),(8). For reuse one case the worst SIR is given in terms of inner cell radius R_{in} are calculated by substituting $(R_{in}, 0)$ in (2), (3),(4),and (5). By equalizing the worst SIR in the two cases, we will have an equation of one unknown which is the inner radius R_{in} as shown in (10, 11) for FRF=3,4 respectively. We assume that the worst SIR is the same for inner and exterior users.

$$SIR \ corner = SIR \ edge \ of \ reuse \ 1 \tag{9}$$

$$\frac{1}{SFRF3(\frac{\sqrt{3}}{2},\frac{1}{2})} = \frac{Rin^{-\alpha}}{S1(Rin,0)}$$
(10)

$$\frac{1}{SFRF4(\frac{\sqrt{3}}{2},\frac{1}{2})} = \frac{Rin^{-\alpha}}{S1(Rin,0)}$$
(11)

SIR is an important parameter in wireless systems because it reflects user's throughput and QOS. The throughput for user i is given in (12) in bits/sec, where $A = \frac{-1.5}{\ln (5BER)}$ [13]. The cell throughput is the aggregate data rate for all users throughput inside the cell and given by (13) [13]. High SE is obtained by maintaining high SIR in the system.SE for user i is shown in equation (14). A user's quality of service can be measured by his outage probability μ which defined as the probability that a user's SIR falls below certain threshold value SIR_{th}, which is given by (15)[22].

$$Ri = \left(\frac{1}{T}\right) \log 2(1 + A SIRi) \tag{12}$$

$$Rcell = \sum_{i=1}^{N} Ri = \left(\frac{1}{T}\right) log 2 \prod_{i} (1 + A SIRi)$$
(13)

$$SE_i = A \log_2 \mathbb{I} (1 + SIRi)$$
(14)

$$\mu_i = prob(SIR_i \le SIR^{th}) = 1 - \prod_{k \neq i} \frac{1}{1 + \frac{SIR^{th}G_{ik}}{G_{ii}}}$$
(15)

III. SFR CASE

Fig.4 shows two tiers network that uses the concept of soft frequency reuse. In this case there are two different power levels according to the user location if it is CEU or CCU. β is the power ratio of P_0/P_i where P_0 is the power of outer region and P_i is the power of inner region. SIR equation for SFR will be different than equations 1, 2 due to the power level difference. For an edge user y served by base station x the SIR equation in case of SFR will be as following:-



Fig.4. Two tiers network with SFR

$$SIR_{SFR} = \frac{\beta p G_{ii}}{p \sum_{zi} G_{ij} + \beta p \sum_{zo} G_{ij}}$$
(16)

where G_{ii} is the path loss gain of the desired signal and G_{ij} is that of the interfering signals, zi is the set of all interfering BSs transmitting to CCUs on the same sub band as user y. zo is the same of zi but for CEUs that use the same frequency of user y. Normalized SIR due to cell radius R is previewed in (17).

$$sir_{SFRCEU} = \frac{\beta(x^2 + y^2)^{-\alpha/2}}{Ii(x, y) + \beta Io(x, y)}$$
(17)
where

wnere

$$li(x,y) = \left[\left(x \pm \sqrt{3} \right)^2 + y^2 \right]^{-\frac{\alpha}{2}} + \left[(x \pm 2\sqrt{3})^2 + y^2 \right]^{-\alpha/2} \\ + \left[(x \pm \sqrt{3}/2)^2 + (y \pm 3/2)^2 \right]^{-\alpha/2} \\ + \left[(x \pm \sqrt{3})^2 + (y \pm 3)^2 \right]^{-\alpha/2}$$
(18)

and

$$Io(x,y) = [(x \pm 1.5\sqrt{3})^{2} + (y \pm 1.5)^{2}]^{-\alpha/2} + [x^{2} + (y \pm 3)^{2}]^{-\alpha/2}$$
(19)

For an CEU the final expressions of outer and inner interference are:

$$Io = 2^{-\alpha} + 4^{-\alpha} + 2\left(7^{\frac{-\alpha}{2}} + 13^{\frac{-\alpha}{2}}\right)$$
(20)

$$Ii = 2\left[1 + 2^{-\alpha} + 2 * 7^{-\frac{\alpha}{2}} + 19^{-\frac{\alpha}{2}} + 13^{-\frac{\alpha}{2}}\right]$$
(21)

On the other hand for CCU located at (x, y) and uses f2, its worst SIR changes to (22). Equations (18, 19) will be changed to (23, 24) due to the change of interfering BSs. See table I last two columns and Fig.4.

$$sir_{SFRCCU} = \frac{(x^2 + y^2)^{-\alpha/2}}{Iic(x, y) + \beta Ioc(x, y)}$$
(22)

$$lic(x,y) = \left[\left(x + \sqrt{3} \right)^2 + y^2 \right]^{-\frac{\alpha}{2}} \\ + \left[\left(x - 2\sqrt{3} \right)^2 + y^2 \right]^{-\frac{\alpha}{2}} \\ + \left[\left(x \pm 1.5\sqrt{3} \right)^2 + \left(y \pm \frac{3}{2} \right)^2 \right]^{-\frac{\alpha}{2}} \\ + \left[\left(x + \sqrt{3} \right)^2 + \left(y \pm 3 \right)^2 \right]^{-\frac{\alpha}{2}} \\ + \left[x^2 + \left(y \pm 3 \right)^2 \right]^{-\alpha/2}$$
(23)

$$loc(x,y) = \left[(x+0.5\sqrt{3})^2 + (y\pm1.5)^2 \right]^{-\frac{\alpha}{2}} + \left[(x-\sqrt{3})^2 + (y\pm3)^2 \right]^{-\frac{\alpha}{2}} + \left[(x-\sqrt{3})^2 + y^2 \right]^{-\frac{\alpha}{2}} + \left[(x+2\sqrt{3})^2 + y^2 \right]^{-\frac{\alpha}{2}}$$
(24)

For optimum inner radius analysis in case of SFR the worst SIR of CCU at (0,1). This is done by equalizing SIR as follows.

$$sir_{SFRCEU}(0,1) = sir_{SFRCCU}(0,r)$$
(25)

and using (17) and (22) in (23) to solve for r

IV. NUMERICAL RESULTS

Fig 5 shows the relation between CEU SIR and α for different possible values of α . The relation are drawn for the two FFR cases using equations (4,5) and (6) and also for SFR case at different power ratios 1,2,4,8,200 using equations (17,20 and 21). Practically the threshold of SIR should be lower or equal to SIR corner. From the figure it is clear that generally Worst SIR increases with the path loss exponent for the two FFR cases and SFR cases. As α increases the path loss gain increase and hence the signal value increase to substitute the increased losses. The best SIR here is the case of FFR=4. This due to the reduced interference resulted from FFR=4. The smallest SIR value is at SFR with $\beta=1$ due to large interference value resulted at the edge because of using reuse one. For β =200 SFR curve it is close to FRF=3 curve this is because when $\beta=1$ SFR turns to reuse 1 and when $\beta=\infty$ it turns to FFR=3.



Fig.5 SIR worst Vs Path loss exponent for FFR=3, FFR=4, SFR at β =1, 2, 4, 8, and 200 respectively

Fig 6 shows the relation between the optimum inner radius and path loss exponent for R=1km. The curves are drawn using equations (10) and (11) for FRF=3, and FRF=4 cases, and using (25) for SFR case with different power ratios. Compared to [12, 13] for α =3.6 R_{opt}=560 m for FRF=3. It is near 600 m, but here we are using practical SIR threshold value greater than 10 dB For the same value of α R_{opt}=480 m for FRF=4. In case of SFR the largest inner radius results from β =1 while the worst case is at β =200. A practical inner radius values resulted for β =2, and 4.

Generally for the same value of path loss exponent the optimum inner radius of FRF=3 is larger than optimum radius of FRF=4. This is due to the Worst SIR in FRF=4 is larger than Worst SIR in FRF=3 for the same path loss exponent.



Fig.6 path loss exponent Vs Optimum radius for FRF=3, FRF=4, and SFR for $\beta{=}1,\,2{,}4{,}8{,}$ and 200

Worst SIR in case of FRF=4 is larger than FRF=3. This means that FRF=3 is better than FRF=4. Fig.7 shows SE of edge user with path loss exponent in the two different FFR cases and SFR case for different power ratios. SE is calculated using 10^{-5} bit error rate. The best SE is at FRF=4 and the worst one is at SFR with β =1;



Fig.7 SE Vs Path loss exponent for FRF=3, FRF=4, and SFR with β =1,2,4,8, and 200.

V. CONCLUSION

Frequency reuse is one of the most important ICI mitigation techniques. It is\divided mainly to hard, FFR, and SFR Calculating worst SIR of the system is very valuable. The problem of optimizing the inner distance has been solved by making use of worst SIR. The approach was applied to two different FFF cases FRF=3 and FRF=4. It was found that the worst SIR depends basically on the Path loss exponent. For the same Path loss exponent FRF=4 has larger worst SIR than FRF=3. FRF=3 has larger optimum radius than FRF=4 for the same path loss exponent. For SFR practical values of β should be 2 or 4. There is a tradeoff between FFR and SFR for better QOS or better capacity respectively.

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