Performance Analysis of Uplink Fractional Frequency Reuse using Worst Case Signal to Interference Ratio

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***Abstract*—Fractional Frequency Reuse (FFR) is an efficient mitigation technique for modern cellular networks because of its low complexity and coordination requirements and resource allocation flexibility. This work considers the use of FFR in the cellular uplink where we analyze the uplink worst case Signal to Interference Ratio (SIR) for two main FFR schemes; which are strict FFR and Soft FFR. A closed form expression is derived analytically for the worst SIR and for the best inner radius. This analytical technique is utilized to configure a FFR solution for the uplink of OFDMA cellular system. The analysis is performed using two-tier cellular network with uniform user density and for three different cases of FFR. The effect of power control exponent on both FFR schemes is also studied. The inner radius configuration depends on equalizing the worst SIR for both inner and outer edges of the cell. Numerical results show that FFR with reuse four yields the highest SIR. It is noticed that power control exponent doesn't affect strict FFR but affects SFR as it reduces its SIR and inner radius.**

Keywords—Wireless communications; inter cell interference; fractional frequency reuse; power control

I. INTRODUCTION

Wireless cellular networks are evolving from voice-oriented to ubiquitous mobile- broadband data networks. So as to meet these growing requirements, 3GPP Long Term evaluation (LTE) [1] targets a spectrum reuse factor of one to achieve higher system capacity and spectrum efficiency. However, reuse one leads to strong Inter Cell Interference (ICI) by the users that use the same frequency in neighboring cells in these Orthogonal Frequency Division Multiplexing systems (OFDMA) [2, 3]; where the strongest interference affects users at the cell edges. Effective Inter-Cell Interference Coordination (ICIC) and spectrum utilization management is an important and challenging issue for wireless communication systems design.

Uplink performance analysis is highly recommended due to symmetric traffic applications like video-calls, social networking, and real time multimedia applications. Compared to downlink analysis uplink analysis, it is more difficult as it faces three main constraints [4].

The first one is that interference in downlink analysis comes from fixed locations, while in uplink; interference is introduced by mobile devices across the network. The second constraint is that uplink analysis uses location dependent power control that makes the transmit power highly variable; hence significant changes of interference statistics compared to the downlink. The third constraint is that maximum power and average transmit power are important for the life time of battery powered mobile devices. Thus, modeling ICIC techniques in the uplink is challenging due to modeling the varying interference generated by distributed mobile devices with limited transmit power and highly variable transmit power due to power control.

This paper focuses on one particular ICIC technique known as FFR. Strict FFR balances the link throughput across the coverage area where Cell Center Users (CCUs) use universal frequency reuse and Cell Edge Users (CEUs) use frequency reuse larger than one. FFR was first proposed for GSM networks [5]; Fig. 1 illustrates the idea of FFR with reuse three, where R_{in} defines the interior region area illustrated by yellow circles.$R_{ext} - R_{in}$ defines the exterior region illustrated by no, solid and dashed lines cells in Fig 1.a.

The bandwidth is divided into two main parts; the first part is assigned for reuse one and the second part are divided equally into three parts for reuse three. Generally, there are two primary schemes of FFR: strict FFR and Soft Frequency Reuse (SFR) [6]. For SFR, the overall bandwidth is shared by all base stations and the bandwidth is divided into major and minor sub bands [6]. Major sub band is used for both center and edge users, while minor sub band is used only for center users. Transmission power for major sub-band is larger than minor sub band power; SFR with reuse three is shown in Fig.1.b.

The difficulty of modeling the cellular uplink arises from the correlation between mobile and base station locations, transmit power, and distribution of the resulting interference. One famous analytical approach is the Wyner model [7] that is primarily used for CDMA networks. This model assumes constant average ICI which is not an exact assumption for OFDMA based networks [8]. A recent work for modeling the uplink based on stochastic geometry is presented in [9], the model uses approximations for grid and Poisson point process models to develop expressions for spectral efficiency.

In addition to the spatial distribution of base stations and mobiles, another modeling challenge arises from the use of open and closed loop fractional power control techniques [10]. In [11], an analytical model based on fluid model [12] is proposed.

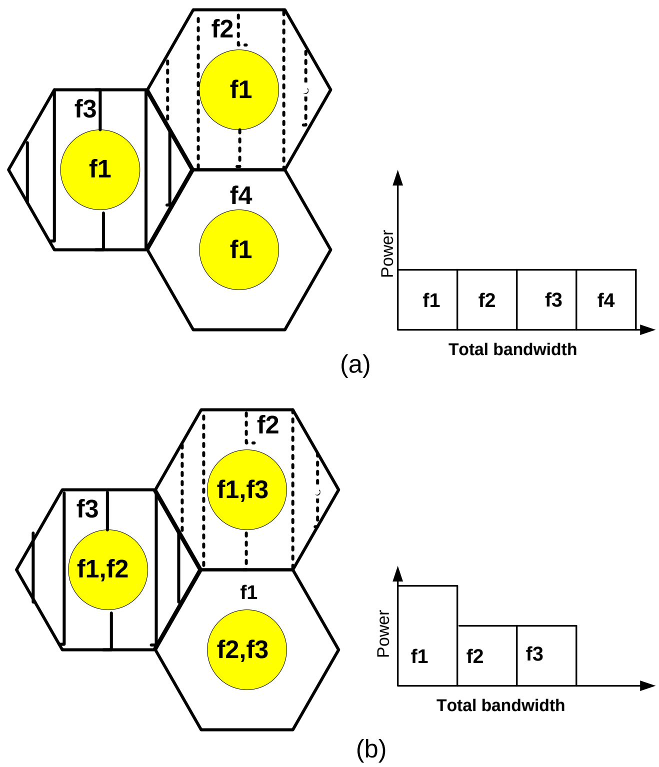

Fig. 1. FFR Concept. a) Strict FF. b) SFR.

The model is based on approximations for hexagonal grid based base station locations to derive analytical expressions for SINR and spectral efficiency of CEUs and CCUs.

In [13], the authors consider a hexagonal network with two tiers of interference; they used Markov chains to model the loads of the different base stations which allow iterative calculation of ICI. Also they have provided numerical simulation results to show the tradeoff problem of static FFR between improvement in the edge, user outage, and sum-rate loss compared to universal reuse. In [14], the authors model the network as a hexagonal grid and calculate user's SINR as a function of their locations.

Performance of SFR in the uplink have been studied using system simulations based on hexagonal grid network with putting into consideration transmit power limitations. In [15], the authors tried to solve the uplink ICIC problem by using a decentralized adaptive soft frequency reuse scheme; where they have used physical resource block (PRB) reuse avoidance/minimization and cell edge bandwidth breathing to solve the problem.

The authors in [16] investigated the performance of the OFDMA uplink with FFR schemes; they have shown that SFR is well performing in the uplink as it does not depend on resource coordination. Network parameters like user distribution and the number of tiers of interfering mobiles are effective parameters that uplink analysis depends on [17]. In [18], the authors used a traffic-adaptive SFR technique for the uplink; where they showed the performance gains for interior and edge users' throughput by using system simulations of hexagonal base stations grid aided by 3GPP standard body.

In this paper, a closed form solution for not only worst case SIR but also for optimum inner radius in OFDMA uplink cellular system is derived. A new method to compute the inner region radius based on uplink worst SIR value, in order to clarify inner radius that equalizes worst edge SIR on both edges of exterior and interior regions, is proposed. The paper is organized as follow: section two details the system model and derives the worst SIR and optimum inner radius for FFR with reuse three. Section three repeats section two for FFR value of four. Section four, previews the same analysis for SFR. Numerical simulation results are provided in Section five.

II. SYSTEM MODEL FOR FFR3

We consider the uplink of an OFDMA cellular network with two-tier, and utilize FFR with reuse three as shown in Fig.2, where each BS is equipped with omnidirectional antenna. The distance between any two adjacent BSs is $2d$ where $d = \frac{\sqrt{3}}{2}R$ where R is the cell radius. Interfering mobiles are given by the set K and the distance between an interfering mobile i and home BS is defined asD_i. The distance of an interfering mobile to its serving BS is denoted by R_i. The closest interfering mobile must be served by a different cell and cannot be closer to the home cell than the transmitting mobile which means $D_i > R_i$. Channel variations due to Path loss is considered, where we use the exponent path loss model given by $G = G_0 r^{-\alpha}$, where α is the path loss exponent [19]. The parameter r is the distance between the BS and its serving mobile User Equipment (UE). The constant G_0 is given by $G_0 = (\frac{C}{4\pi f})^2$ where f is the center frequency and C is the speed of light. Each sub band exposes to exponentially distributed fast fading power g with unit mean and Gaussian noise with power σ^2.

The transmit powers used by mobiles rely on fractional path loss inversion of the form $Pr^{\alpha\mu}$, where $\mu \in [0,1]$ is the power control exponent and P is the fixed received power for all mobiles. One specific formula of P that is used by 3GPP standards like LTE is given as [20]:

$$P = \mu(SNR_0 + \sigma^2) + P_{max}(1-\mu) \quad (1)$$

Where P_{max} is the maximum transmit power, σ^2 is the noise power, and SNR_0 is the target threshold in dB.

The uplink power control strategy of the mobiles is as follow: when $\mu = 0$ all mobiles transmit with constant power $P = P_{max}$. This occurs when the mobile is far away from its serving BS.While selecting $\mu = 1$ leads to desired signal received power at the base station of gP regardless of the user location.

The SIR is given by

$$SIR = \frac{gPr^{\alpha(\mu-1)}}{I_K} \quad (2)$$

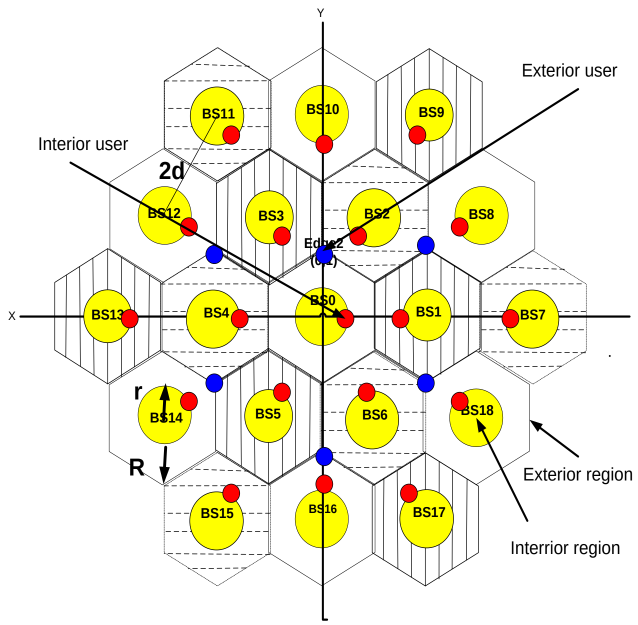

Fig. 2. Two-tier network with FFR three

Where I_I for a given set of interfering mobiles K is given by

$$I_K = \sum_{i \in K} g_i P (R_i^{\alpha})^{\mu} D_i^{-\alpha} \quad (3)$$

If a user lies uniformly in $U_i(X,Y)$ at a distance $r = \sqrt{X^2+Y^2}$ from BS0 (see Fig.2), upon its location this user can be grouped either inner or exterior user. So as to compute worst case SIR for inner user we consider 18 interfering inner mobile users, shown as red dots in Fig.1, on that user lies in the inner regions of BS1:18 respectively.

$$SIR_{in} = \frac{gPr^{\alpha(\mu-1)}}{gPr^{\alpha\varepsilon}[6(2d-r)^{-\alpha}+6(4d-r)^{-\alpha}+6(3R-r)^{-\alpha}} = \frac{r^{-\alpha}}{6\left(\left(\sqrt{3}R-r\right)^{-\alpha}+\left(2\sqrt{3}R-r\right)^{-\alpha}+(3R-r)^{-\alpha}\right)} \quad (4)$$

For exterior user lies at one of the corner edges of the home cell, we consider six nearest interfering mobile users that use the same frequency and lie at the closest distance to BS0 as shown in Fig.2 (blue dots). Thus a general SIR expression for exterior user uplink SIR is as follow:

$$SIR_{FFR3} = \frac{gPR^{\alpha(\mu-1)}}{gP(R^{\alpha})^{\mu} * 6(2R)^{-\alpha}} = \frac{2^{\alpha}}{6} \quad (5)$$

From (4) and (5) the worst SIR depends basically on path loss exponent α. In order to find the best FFR inner radius, our technique calculates the worst SIR for inner and exterior users. The worst SIR for exterior user that utilizes FFR with reuse three is calculated and given in (5). For reuse one case the worst SIR is given in terms of inner cell radius r is calculated in (4). By equalizing the worst SIR in the two cases, the result is an equation of one unknown which is the inner radius r as shown in (7).

$$SIR_{in} = SIR_{FFR3} \quad (6)$$

$$\frac{r^{-\alpha}}{(\sqrt{3}R-r)^{-\alpha}+(2\sqrt{3}R-r)^{-\alpha}+(3R-r)^{-\alpha}} = 2^{\alpha} \quad (7)$$

High SE is obtained by maintaining high SIR in the system. SE for user i is shown in Eq (8), where **Ω** is the inverse of the reuse factor.

$$SE_i = \Omega \log_2 (1 + SIR_i) \quad (8)$$

A user's QOS can be measured by his outage probability β which defined as the probability that a user's SIR falls below certain threshold value SIR^{th}, and is given by [10,15]

$$\beta_i = prob(SIR_i \le SIR^{th}) = 1 - \prod_{i \in K} \frac{1}{1+SIR^{th}\frac{r_i^{-\alpha}}{r^{-\alpha}}} \quad (9)$$

III. FFR WITH REUSE FOUR CASE

For FRF=4 case, the exterior region uses reuse three as the total bandwidth equals $BW_{tot} = B_1 + 4B_2$.Where B_1 is the BW allocated to reuse one region and the rest of the BW is divided equally to four equal. For inner mobile user case, its worst SIR will be the same of FRF=3 case as in (4). For exterior mobile user case, using the same previous analysis of FRF=3, its worst SIR is given by

$$SIR_{FFR4} = \frac{gPR^{\alpha(\mu-1)}}{gP[4(R^{\alpha})^{\mu}\left(\sqrt{7}R\right)^{-\alpha} + 2\left(\frac{\sqrt{3}R}{2}\right)^{\alpha\mu} (1.5\sqrt{3}R)^{-\alpha}} = \frac{1}{4(7)^{-\alpha/2} + 2(1.5\sqrt{3})^{-\alpha}} \quad (10)$$

For best inner radius in this case (7) is changed to

$$\frac{r^{-\alpha}}{3[(\sqrt{3}R-r)^{-\alpha}+(2\sqrt{3}R-r)^{-\alpha}+(3R-r)^{-\alpha}]} = \frac{1}{2(7)^{-\alpha/2}+(1.5\sqrt{3})^{-\alpha}} \quad (11)$$

IV. SOFT FREQUENCY REUSE CASE

Fig.3 shows two tiers network that uses the concept of soft frequency reuse. In this case, there are two different power levels according to the mobile user location if it is CEU or CCU. β is the power ratio of $\frac{P_o}{P_i}$where P_o is the transmitted power of exterior mobile user to its serving BS and P_i is the transmitted power of interior user to its serving BS. ICI comes from both interior and corner uplinks. As shown in Fig.1.b the power control factor $\beta \ge 1$ is applied. Interior user transmit power will equal $Pr^{\alpha\mu}$. A practical range of β is between 2 and 100 [20, 21]. SIR equation for SFR will be different than FFR three and four cases because of the power level difference.

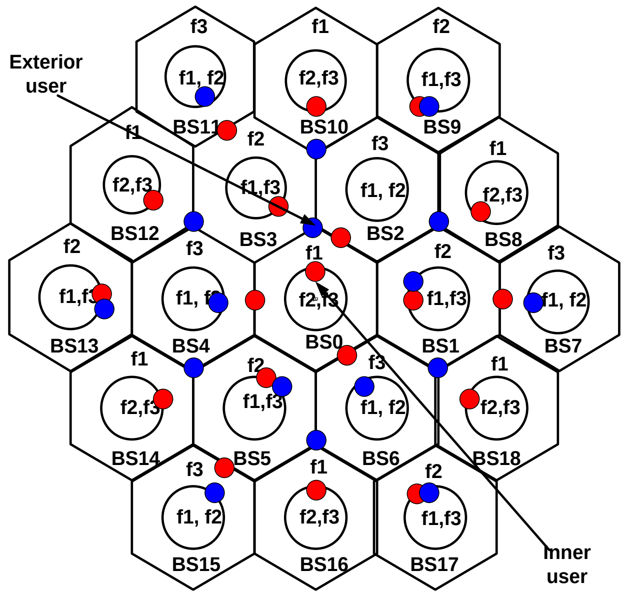

Fig. 3. Two tiers network with SFR

For an interior user y served by home base station BS0 and far away from it by distance r. The SIR equation in case of SFR will be as following:-

$$SIR_{SFRinner} = \frac{gPr^{\alpha(\mu-1)}}{P\sum_{z\in Z_{inner}} g_z r_z^{\alpha\mu} D_z^{-\alpha} + \beta p \sum_{z\in Z_{edge}} g_z R_z^{\alpha\mu} D_z^{-\alpha}} \quad (12)$$

where, Z_{inner} is the set of all interfering mobile stations transmitting to its serving base stations on the same sub band as user y. Z_{edge} is the same of z_{inner} but for edge mobile users that use the same frequency of user y. in this case SIR will equal

$$SIR_{SFRinner} = \frac{r^{\alpha(\mu-1)}}{3\beta(\frac{\sqrt{3}}{2}R^{\alpha})^{\mu}\left[(\frac{\sqrt{3}}{2}R)^{-\alpha}+\left(\frac{3\sqrt{3}R}{2}\right)^{-\alpha}\right]+3(r^{\alpha})^{\mu}[(\sqrt{3}R-r)^{-\alpha}+(2\sqrt{3}R-r)^{-\alpha}+2(3R-r)^{-\alpha}]} \quad (13)$$

If we define X as the ratio of inner and outer radius of the cell i.e $x=\frac{r}{R}$ then Eq.13 will equal

$$SIR_{SFRinner} = \frac{x^{\alpha(\mu-1)}}{3\beta(\frac{\sqrt{3}}{2})^{\alpha\mu}\left[(0.5\sqrt{3})^{-\alpha}+(1.5\sqrt{3})^{-\alpha}\right]+3x^{\alpha\mu}[(\sqrt{3}-x)^{-\alpha}+(2\sqrt{3}-x)^{-\alpha}+2(3-x)^{-\alpha}]} \quad (14)$$

For exterior mobile user its SIR will equal

$$SIR_{SFRouter} = \frac{g\beta PR^{\alpha(\mu-1)}}{P\sum_{z\in Z_{inner}} g_z r_z^{\alpha\mu} D_z^{-\alpha} + \beta p \sum_{z\in Z_{edge}} g_z R_z^{\alpha\mu} D_z^{-\alpha}} =$$

$$\frac{\beta R^{\alpha(\mu-1)}}{\beta(R^{\alpha})^{\mu}[6(2R)^{-\alpha}]+6(r^{\alpha})^{\mu}\left[(\sqrt{3}R-r)^{-\alpha}+(2\sqrt{3}R-r)^{-\alpha}\right]} = \frac{\beta}{6x^{\alpha\mu}\left[(\sqrt{3}-x)^{-\alpha}+(2\sqrt{3}-x)^{-\alpha}\right]+6\beta 2^{-\alpha}} \quad (15)$$

The best inner radius r is calculated by equalizing Eqs (14), and last step of Eq (15). This yields an equation of one unknown which is r.

V. NUMERICAL RESULTS

Fig. 4 shows the relation between SIR and α for different possible values of α using (4), (5), and (10) of reuse one, FRF=3, and FRF=4 respectively. Practically the SIR at different locations is greater than worst SIR at the cell corner. It is noted that as α increases, the worst SIR also increase. This is because as α increases (attenuation increases) both the received signal and interfering signals powers decrease. However the decrease in the interfering signal is more than that of received signal resulting in an increase of SIR. It is clear that the power control exponent μ has no effect on these three FFR schemes according to their corresponding equations. For the same value of α, FFR with reuse four has the largest SIR value. It is obvious from the curve that reuse one has lowest values of SIR due to large interference at the edges. For $\alpha = 3.5$, the worst SIR= -7, 4, and 6dB for reuse one,FRF=3,andFRF=4respectively.

Fig. 5 previews worst SIR of SFR against the ratio of inner radius to cell radius at different values of power control factor β which are 2, 4, 8, 10, and 100. The figure is drawn using equation (15) for α=4.The relation is drawn twice for no power control exponent ($\mu = 0$) represented by solid lines, and $\mu = 0.6$ represented by dotted lines. The effect of power control on mobile is obvious from the figure as the SIR increases in case of $\mu = 0$ than no power exponent case. The best SIR occurs for $\beta = 100$ and $\mu = 0.6$. While the lowest SIR value occurs when $\beta = 2$ and $\mu = 0$. SIR is proportional to both values of β and μ. For larger values of (r/R) SIR degrades quickly due to the increased ICI resulted from neighboring cells.

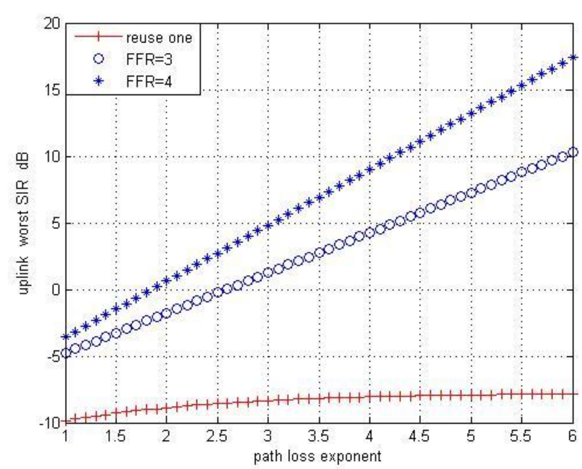

Fig. 4. worst SIR vs. path loss exponent for reuse1, FFR3, and FFR4

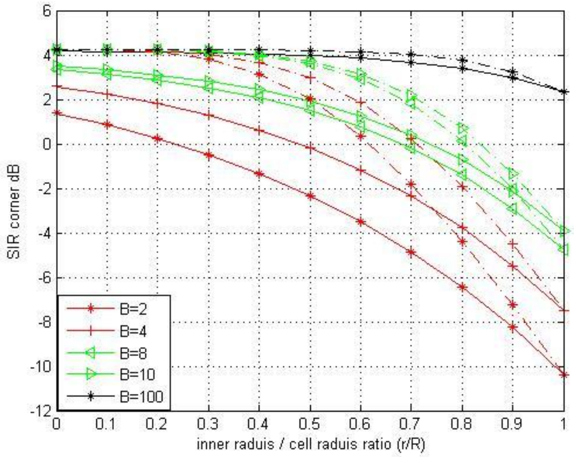

Fig. 5. worst SIR vs. path loss exponent for SFR at β=2, 4, 8, 10, and 200. α=4, no power control μ=0(lines), with power control μ=0.6(dotted lines)

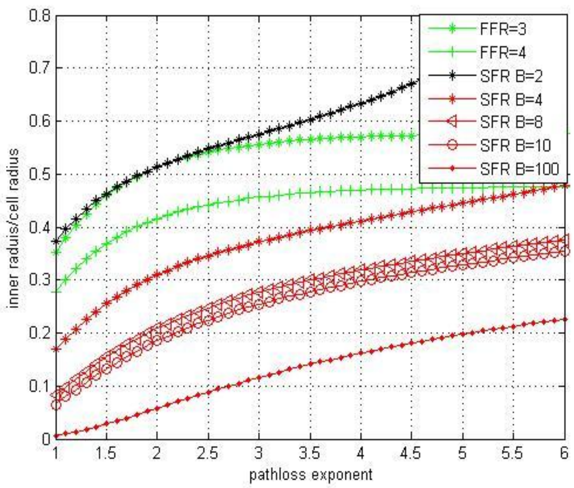

Fig. 6. Inner radius to cell radius ratio vs. path loss exponent at no power control (μ=0)

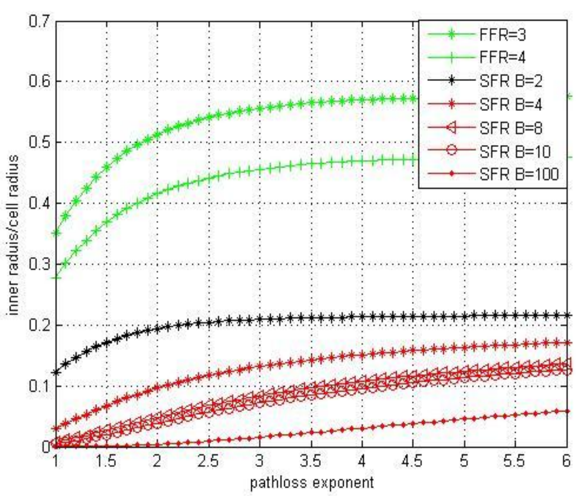

Fig. 7. Inner radius to cell radius ratio vs. path loss exponent at power control (μ=0.6)

Figs 6, 7 present the relation between the ratio of inner radius to cell radius versus path loss exponent for two main types of FFR with and without power control exponent respectively. Fig.6 shows the no power control case where μ = 0. It is clear that strict FFR curves do not change in the two figures due to the negligible effect of μ. On the contrary for SFR case power control exponent reduces the overall ratio of inner to cell radius for the same path loss exponent as shown in the two figures.

VI. CONCLUSIONS

This work has presented anew analytical method for analyzing worst case SIR and best inner radius in the uplink of cellular networks utilizing FFR schemes. These expressions are based on the distribution of the mobile users in two tiers network so as to compute the worst SIR. The problem of calculating the universal reuse inner distance is solved by equalizing worst SIR of both inner and outer regions. The approach was applied to three different FFR cases FRF=3, FRF=4 and SFR. The effect of power control exponent was also studies on the two main FFR schemes as it was found that it affects SFR greatly and has no effect on strict FFR.

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