ERROR EXPONENTS FOR DISTRIBUTED DETECTION WITH FEEDBACK[1]

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Abstract

We investigate the effects of feedback on a decentralized detection system consisting of N sensors and a detection center. It is assumed that observations are independent and identically distributed across sensors, and that each sensor compresses its observations into a fixed number of quantization levels. We consider two variations on this setup. One entails the transmission of sensor data to the fusion center in two stages, with broadcast of feedback information from the center to the sensors after the first stage. The other variation involves information exchange between sensors prior to transmission to the fusion center; this exchange is effected through a feedback center, which processes binary data from the sensors and thereafter broadcasts a single feedback bit back to the sensors. We show that under the Neyman-Pearson criterion, only the latter type of feedback results in an improvement in the asymptotic performance of the system (as $N \to \infty$) and we derive the associated error exponents.

Summary

Consider a detection system consisting of a large number of sensors $S_1, \ldots S_N$ and a detection center C_d. The sensor observations are represented by the discrete random variables $X_1, \ldots, X_N$ taking values in a common alphabet $\mathcal{X}$, and are assumed to be independent and identically distributed. Inherent communication constraints require each sensor S_i to compress its observation X_i into a discrete variable V_i taking values in $\mathcal{V}$, where $|\mathcal{V}| = M_V \leq |\mathcal{X}|$. Upon receipt of $V_1, \ldots, V_N$, the detection center makes a decision as to whether sensor observations are governed by a null univariate distribution P_X or an alternative Q_X.

The above framework is common to a diverse body of literature on distributed detection. In this paper we focus on issues of asymptotic performance as the number N of sensors increases, with emphasis on the effect of introducing feedback and inter-sensor communication. We assume throughout that optimality is assessed in terms of the Neyman-Pearson criterion, i.e., minimization of type II error subject to a fixed upper bound ϵ on the type I error, and that the asymptotic figure of merit is given by the error exponent of the least type II error achievable at level ϵ.

To introduce the problem, we consider the simple (and known) situation in which feedback is altogether absent. Here each sensor independently uses a random encoder $\Delta_{V_i|X_i}$ to generate V_i, and the center decides in favor of the null hypothesis if and only if the received sequence $(V_1, \ldots, V_N)$ lies in some acceptance region $\mathcal{A}_N \subset \mathcal{V}^N$. The error exponent in this case is given for all $\epsilon \in (0,1)$ by

$$\theta^{(1)}(M_V, \epsilon) = \max_{\substack{\Delta_{V|X}: \\ P_V = P_X \Delta_{V|X},\ Q_V = Q_X \Delta_{V|X}}} D(P_V \| Q_V) ,$$

where $P_X \Delta_{V|X}$ is the vector-matrix product of the $|\mathcal{X}|$-dimensional row vector P_X and the $|\mathcal{X}| \times M_V$ stochastic matrix $\Delta_{V|X}$. It also follows that there exist asymptotically optimal systems in which all sensors employ the same encoder $\Delta_{V|X}$, which by virtue of convexity of the divergence functional can be taken as deterministic.

In investigating the effect of feedback on the above system, we first consider the case of two-stage local compression using a fixed-bit feedback packet from the detection center. Here we assume that each sensor generates two messages: U_i in the first stage of the compression, followed by V_i in the second. After the first stage, and based on the received sequence $(U_1, \ldots, U_N)$, the detection center C_d produces a feedback message $Z \in \mathcal{Z}$ which is communicated to the the sensors. The sensors then generate the second message V_i, which is used by the detection center to complete the inference process.

Our result is the following: provided the size of the alphabet $\mathcal{Z}$ is fixed in N, use of Z does not lead to an improvement in asymptotic performance. More precisely, the error exponent $\theta^{(2)}(M_U, M_V, \epsilon)$ resulting from this scheme is achieved by deterministic encoders that are common to all sensors, and satisfies

$$\theta^{(2)}(M_U, M_V, \epsilon) = \theta^{(1)}(M_U \times M_V, \epsilon) .$$

Thus two-stage compression with fixed-bit feedback is asymptotically no better than one-stage compression without feedback.

The above conclusion does not, however, imply that fixed-bit feedback is altogether valueless. To demonstrate this, we consider next a setup in which feedback is used to provide partial information exchange between sensors prior to transmission to the detection center C_d. Specifically, we introduce a feedback center C_f, to which sensors transmit messages U_i. The task of C_f is to generate a nondegenerate feedback variable Z whose alphabet size does not vary with N, based on which the sensors re-encode their observations into messages V_i. The V_i's are then communicated to the detection center C_d, which makes the final decision based *solely* on the sequence $(V_1, \ldots, V_N)$.

Under the assumption that sensors use identical encoders and the constraints $M_U = M_V = 2$ and $D(P_X \| Q_X) < \infty$, we show that the error exponent for this problem is achieved by deterministic encoders, and is given by

$$\theta^{(3)}(2, 2, \epsilon) = \max_{\substack{\Delta_{UV|X}: \\ P_{UV} = P_X \Delta_{UV|X} \\ Q_{UV} = Q_X \Delta_{UV|X}}} \min_{\substack{\tilde{P}_{UV}: \\ \tilde{P}_U = P_U,\ \tilde{P}_V = P_V}} D(\tilde{P}_{UV} \| Q_{UV}) ,$$

where $\Delta_{UV|X}$ is a $|\mathcal{X}| \times 4$ stochastic matrix. It is not difficult to see that $\theta^{(3)}(2, 2, \epsilon)$ is in general greater than $\theta^{(1)}(2, \epsilon)$. We conclude that information exchange between sensors effected by this type of feedback can yield an improvement in asymptotic performance.

1. Research suppørted by the Systems Research Center (a National Science Foundation Engineering Research Center) at the University of Maryland and by the Minta Martin Fund for Aeronautical Research administered by the College of Engineering, University of Maryland.