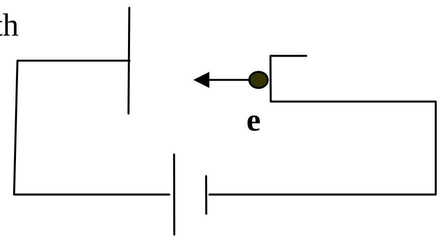
Lecture 10

10.1 Diffraction of X-rays:

The presence of a wave nature is best examined by the known light phenomena, like diffraction, reflection and interference. Consider now an electron being accelerated by falling through a potential difference of 100V.

The De Broglie wavelength associated with this electron can be calculated as follows:



$$\frac{1}{2}mv^2 = qV$$

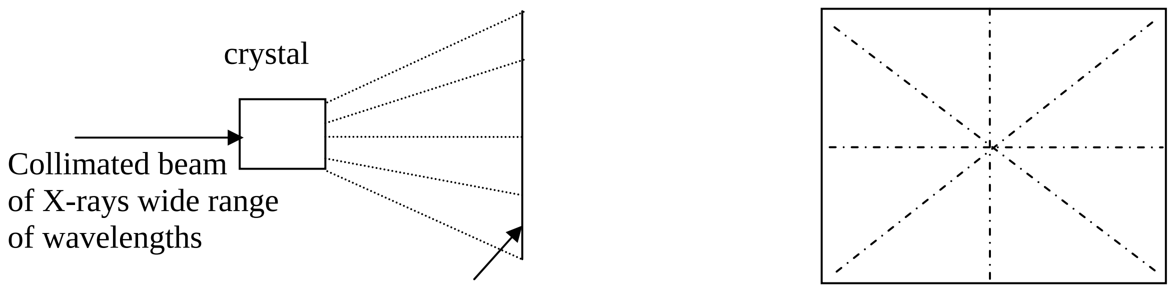
$$v = \sqrt{\frac{2q}{m}V} = 5.9\times10^5\sqrt{V}\,m/s$$

$$v = 5.9\times10^6\,m/s$$

$$\lambda_D = \frac{h}{mv} = \frac{6.626\times10^{-34}}{9.1\times10^{-31}\times5.9\times10^6} = 1.23\overset{o}{A}$$

This wavelength is in the X-rays range. So, to exhibit the light phenomena of the wave associated with the electron, we need an optical grating of a spacing between slits of $1\overset{o}{A}$ which is difficult to obtain $\cong 10\,millions\,slits\,per\,mm$. Diffraction of even X-rays was hard to detect at that time. In 1912, Von Laue discovered that atoms in crystals of molecules are arranged in a very neat and orderly manner. The interatomic spacing falls within the required slit separation of a grating. So, crystals can be used to observe diffraction of X-rays and also of electrons. A main difference is that the crystal is a 3-D grating whose diffraction centers are the atoms.

10.2 Friedrick and Knipping experiment:



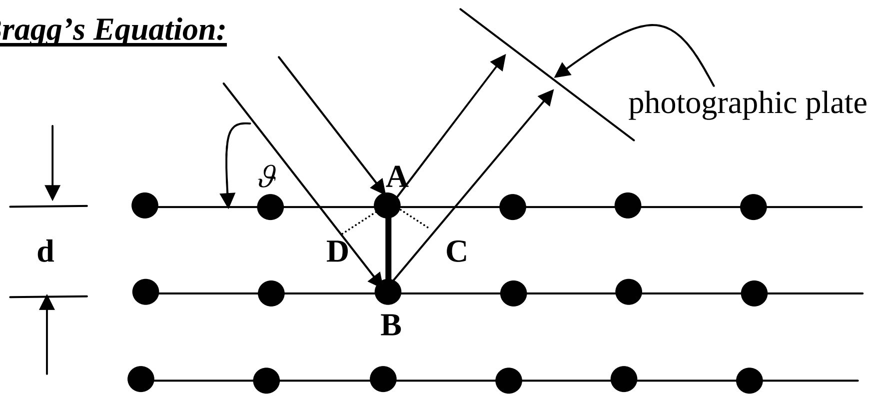
Diffraction pattern

The experiment was performed by allowing an x-ray collimated beam to pass through a crystal structure and be received on a photographic plate. A neat arrangement of spots on the photographic plate was obtained. The obtained arrangement of spots confirms the idea that atoms are arranged in crystals of solids in an orderly manner.

Bragg in 1913 gave a good explanation for the X-ray diffraction in crystals. He assumed the diffracted X-rays to be scattered at different planes of atoms inside crystals. These planes, full of atoms, are known as rich planes or Bragg planes.

Constructive interference of X-rays takes place in certain directions in crystals, while destructive interference takes place in some other directions.

10.3 Bragg's Equation:

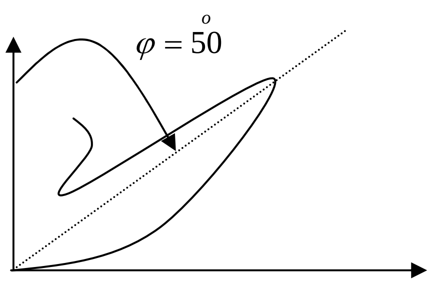


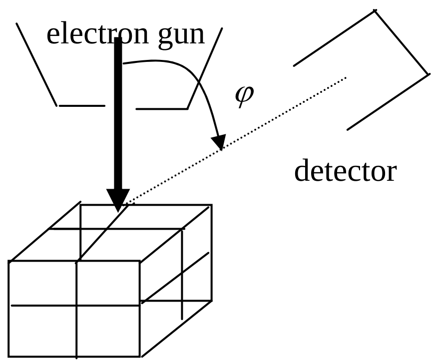
Constructive interference, and hence a bright spot appears when the path difference is equal to nλ .
Path difference = $dB + BC = d\sin\vartheta + d\sin\vartheta$

$$\therefore 2d\sin\vartheta = n\lambda \quad \Rightarrow \; Bragg's\ Equation$$

So, this equation can be used to determine either λ if d is known and ϑ is measured, or to determine the crystal structure if λ is known and ϑ is measured.

10.4 Measurement of electron wavelength, Davison and Germer Experiment:





Nickel Crystal

When an electron beam scatters by a crystal, the scattered beam shows maxima and minima. They found a strong maximum at angle 50^o to the direction of incidence for electrons of energy of 54 eV.
The analysis can be generalized as follows:

$$n\lambda = 2d\sin\vartheta$$
$$= 2d\cos\alpha$$
$$\therefore d = D\sin\alpha$$
$$\therefore n\lambda = 2D\sin\alpha\cos\alpha$$
$$= D\sin 2\alpha$$

Example: from X-Ray diffraction D = 2.15 A^o

$$54 \text{ eV electron } \lambda = \frac{h}{mv} = \frac{h}{m\times 5.9\times 10^5\sqrt{54}} = 1.67\,\overset{o}{A}$$

while for n=1, $\varphi = 50^{o}$, and D = 2.15 A^{o}.

$$\therefore \lambda = 2.15 \sin 50 = 1.64 \overset{o}{A}$$

This compares well with the calculated one (De Broglie). The difference may be attributed to the diffraction of the electron wave taken place at the crystal/air interface.

$$\text{Index of refraction } \mu = \frac{\lambda`}{\lambda} = \frac{n_{ni}}{n_a}$$

Where λ is the wave length inside the crystal and $\lambda`$ is that inside air.

Snell`s law $\mu = \dfrac{\lambda`}{\lambda} = \dfrac{\sin\varphi`}{\sin 2\alpha} = \dfrac{n_{ni}}{n_a}$

$\therefore n\lambda` \dfrac{\sin\varphi}{\sin\varphi`} = D \sin\varphi$

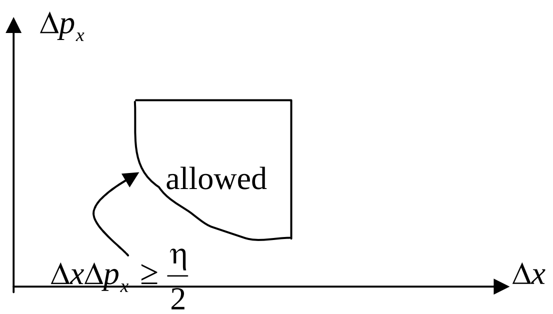
$$n\lambda` = D \sin\varphi`$$

10.5 Heisenberg`s Uncertainty Principle:

Because of the wave nature of material particles, it is hard to tell where the particle is inside the wave packet. All what can be said is that the particle is within this wave packet. The uncertainty principle of Heisenberg refers to the simultaneous determination of the particle's position and momentum. He said that the particle's position on the x-axis can be determined simultaneously with its momentum along the x-axis with certain accuracy as follows:

$$\Delta x \Delta p_x \geq \frac{\eta}{2}$$

where Δx is the uncertainty in position and Δp_x is that of the momentum.



Also if the energy of the particle E at certain time t is to be determined simultaneously, it is obtained with the following uncertainty:

$$\Delta t = \frac{\Delta x}{v}$$
$$E^2 = p^2c^2 + m_o^2c^4$$
$$2E\Delta E = 2p\Delta pc^2$$
$$\Delta E = \frac{p}{E/c^2}\Delta p = \frac{p}{m}\Delta p$$
$$\Delta E = v\Delta p$$
$$\Delta E = \frac{\Delta x}{\Delta t}\Delta p$$

$$\therefore \Delta E\Delta t = \Delta x\Delta p \geq \frac{\eta}{2}$$

10.6 Complementarity Principle

The particle's aspects and wave's aspects complement each other. Both are needed, but can not be observed at the same time. Whether the wave's aspects or the particle's aspects are observed depends on the experimental arrangements.