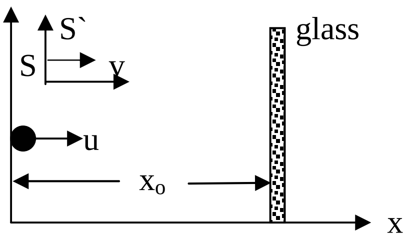
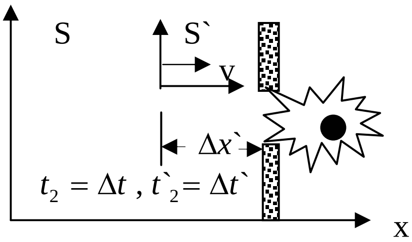
Lecture 5

5.1 Causality





Consider the following two events to take place successively:

Event (1):
S : $x_1 = 0$, $t_1 = 0$
S` : $x`_1 = 0$, $t`_1 = 0$
Event (2):
S decides : $x_2 = x_o$

$$t_2 = \Delta t = \frac{x_o}{u}$$

S`` decides : $x`_2 = \Delta x`$

$$t`_2 = \Delta t`$$

Using Lorentz Transformation

$$t`_2 = \gamma(t - \frac{v}{c^2}x)$$

$$\Delta t` = \gamma(\Delta t - \frac{v}{x^2}x_o)$$

$$\Delta t` = \gamma(\Delta t - \frac{v}{c^2}u\Delta t)$$

$$= \gamma\Delta t(1 - \frac{uv}{c^2})$$

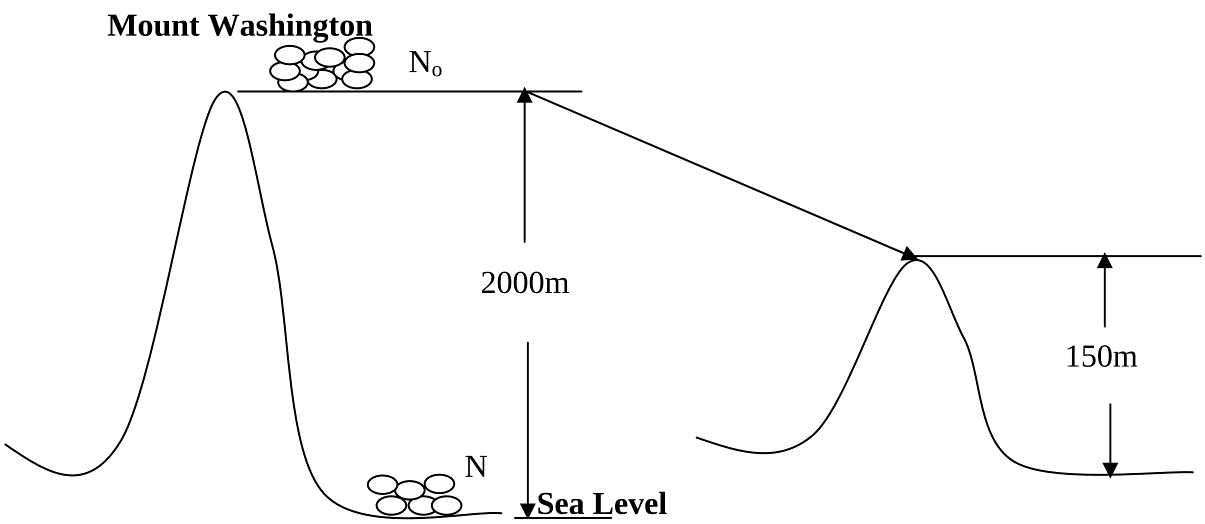
We note the following:

1. As long as v < c and u < c, $\Delta t`$ is positive and the system is **causal.**
2. If v = c and u < c, $\Delta t`$ is still positive and the system is **causal.**
3. If v = c and u = c , $\Delta t` = 0$ and the system is seeing simultaneous events.
4. Only if vu > c^2 (**Impossible**), $\Delta t`$ is negative and the system is **noncausal.**

5.2 Experimental Verification of Lorentz Contraction and Time Dilation:

The first clear example of time dilation was provided over fifty years ago by an experiment detecting muons (m_{muon} = 207 $m_{electron}$.) These particles are produced at the outer edge of our atmosphere by incoming cosmic rays hitting the first traces of air. They are unstable particles, they soon decompose into electrons and neutrino. Their "half-life time" is 1.5 μs, which means that if at a given time you have 100 of them, 1.5 μs later you will have about 50, 1.5 a μs fter that 25, and so on. Anyway, they are constantly being produced many miles up, and there is a constant rain of them towards the surface of the earth, moving at very close to the speed of light. In 1941, a detector placed near the top of Mount Washington (at 2000 meter above sea level) measured about 570 muons per hour coming in. Now these muons are raining down from above, but dying as they fall, so if we move the detector to a lower altitude we expect it to detect fewer muons because a fraction of those that came down past the 2000 meter level will die before they get to a lower altitude detector. Approximating their speed by that of light, they are raining down at $3x10^8$ meter per second, which turns out to be ,conveniently, about 200 meter per μs. Thus they should reach the 300 meter level 1.5 microseconds after passing the 2000 meter level, so, if half of them die off in 1.5 microseconds, as claimed above, we should only expect to register about 570/2 = 285 per hour with the same detector at this level. Dropping another 300 meter, to the 600 meter level, we expect about 280/2 = 140 per hour, at 900 meter about 70 per hour, and at ground level about 6 per hour. (We have rounded off some figures a bit, but this is reasonably close to the expected value).

To summarize: given the known rate at which these raining-down unstable muons decay, and given that 570 per hour hit a detector near the top of Mount Washington, we only expect about 35 per hour to survive down to sea level. In fact, when the detector was brought down to sea level, it detected about 400 per hour! How did they survive? The reason they didn't decay is that in their frame of reference, much less time had passed. Their actual speed is about 0.997c, corresponding to a time dilation factor of about 13.3, so in the 6.67 μs trip from the top of Mount Washington to sea level, their clocks register only 6.67/13.3 = 0.57 μs. In this period of time, only about one-third of them decay.



The equation of muon decay, which gives the number of muons N at any given time t starting from an intial number N_o, is approximated by:

$$N = N_o e^{-t/\tau}$$

Where τ is the half-life time of muons.

If we assume muons to travel with the light velocity c, it take a time $t = \frac{2000}{c} = 6.67\mu s$ to reach the sea level as determined by a stationary observer on the earth. The number of muons expected to reach the sea level can be calculates as:

$$N = N_o e^{-6.67/1.5} = 0.0117 N_o$$

The detector at the sea has detected a number $N=0.714N_o$. So in the muons frame a much lesser time has elapsed, this can be determined as follows:

$$N = 0.714 N_o = N_o e^{-t`/1.5}$$

$$which\ gives\ \ t` = 1.5\ln\frac{1}{0.714} = 0.5\mu s$$

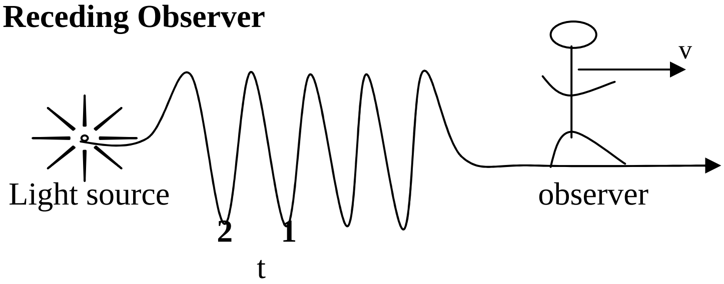
$$\therefore \gamma = \frac{t}{t`} = \frac{6.67}{0.5} = 13.34\,\mu s = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

which gives v = 0.997c. So, from the muon's point of view the earth's surface is approaching the muon with a velocity 0.997c.
This as well could be thought of as that the distance traveled by muons has contracted to the length $L = \frac{L_o}{\gamma} = \frac{2000}{13.34} \cong 150m$, a distance that the approaching earth covers in $0.5\,\mu s$.

5.3 Doppler Effect

This is a particular example of the Doppler effect, first discussed in 1842 by the German physicist Christian Doppler. There is a Doppler Effect for sound waves too. Sound is generated by a vibrating object sending a succession of pressure pulses through the air. These pressure waves are analogous to the flashes of light. If you are approaching a sound source you will encounter the pressure waves more frequently than if you stand still. This means you will hear a higher frequency sound. If the distance between you and the source of sound is increasing, you will hear a lower frequency. This is why the note of a jet plane or a siren goes lower as it passes you. The details of the Doppler Effect for sound are a little different than those for light, because the speed of sound is not the same for all observers - it's 330 meters per second relative to the air, while the speed of light is constant for all observers.

A- Receding Observer



The observer in a time t travels a distance vt away from the source. The light source from a given tick must take $\frac{vt}{c}$ longer time to reach the observer.

$$t_{observer} = t + \frac{vt}{c} = \gamma T_o + \frac{v\gamma T_o}{c}$$

$$T = \gamma T_o \ (1 + \frac{v}{c})$$

$$\therefore \upsilon = \upsilon_o \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \quad < \ \upsilon_o \quad red\ shift.$$

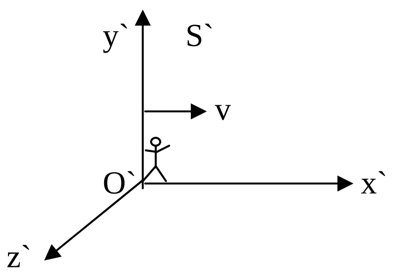
So the light becomes redder.

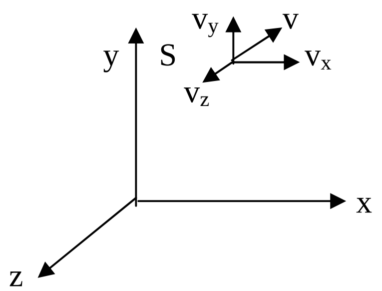
B- Approaching Observer 

We just replace every v with –v.

$$\therefore \upsilon = \upsilon_o \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \quad > \ \upsilon_o \quad less\ redder.$$

5.4 Relativistic Velocity





In the S-frame:

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt} \ \ and \ \ v_z = \frac{dz}{dt}$$

Now, what these velocity components are as determined by the observer of the S`-frame.

Using Lorentz transformation:

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{vx}{c^2})$$

$$dx' = \gamma(dx - vdt)$$

$$dt' = \gamma(dt - \frac{vdx}{c^2})$$

$$v'_x = \frac{dx'}{dt'} = \frac{dx - vdt}{dt - \frac{vdx}{c^2}} = \frac{v_x - v}{1 - \frac{vv_x}{c^2}}$$

Similarly we can get v'_y and v'_z:

dy` = dy

dz` = dz

$$\therefore v`_y = \frac{v_y \sqrt{1-\frac{v^2}{c}}}{1-\frac{vv_x}{c^2}}$$

$$v`z = \frac{v_z \sqrt{1-\frac{v^2}{c^2}}}{1-\frac{vv_x}{c^2}}$$

Inverse Transfromation: Just replace every v with –v and switch the prime.

$$v_x = \frac{v_x + v}{1+\frac{vv`_x}{c^2}}$$

$$v_y = \frac{v`_y \sqrt{1-\frac{v^2}{c}}}{1+\frac{vv`_x}{c^2}}$$

$$v_z = \frac{v`_z \sqrt{1-\frac{v^2}{c^2}}}{1+\frac{vv`_x}{c^2}}$$

<u>5.5 Correspondenc Principle</u>

This principle states that : " In the limit quantum calculations must agree with classical calculations" i. e. there should some limit at which both moder and classical transformations agree and it makes no difference to use either one of them

So, if v is much smaller than c:

$$\therefore \frac{vv_x}{c^2} \quad \& \quad \frac{v^2}{c^2} << 1$$

Henc,

$$v`_x = v_x - v$$

$$v`_y = v_y$$

$$v`_z = v_z$$