1. For the following single-input single-output SS equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t), \qquad \mathbf{x}(0) = \mathbf{x}_0$$
$$y(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t)$$

with $\mathbf{D} \neq 0$, show that the related SS equation

$$\dot{\mathbf{z}}(t) = \left(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\right)\mathbf{z}(t) + \mathbf{B}\mathbf{D}^{-1}v(t), \qquad \mathbf{z}(0) = \mathbf{z}_0$$
$$w(t) = -\mathbf{D}^{-1}\mathbf{C}\mathbf{z}(t) + \mathbf{D}^{-1}v(t)$$

has the property that if the initial conditions of both systems are the same, i.e. $\mathbf{z}_0 = \mathbf{x}_0$, the second system will act as the inverse of the first system, i.e. $w(t) = u(t)$ when $v(t) = y(t)$.