Assume a vehicle that moves in a 2-dimensional plane due to a 2-dimensional applied force. If $p(k)$, $v(k)$ and $f(k)$ are the position of the vehicle, velocity of the vehicle and force applied on the vehicle at time index k, respectively, the discrete time state space representation per dimension of the system is

$$p(k+1) = p(k) + Tv(k)$$
$$v(k+1) = (1-\alpha)v(k) + \frac{T}{m} f(k)$$

where T is the sampling period, m is the mass of the vehicle, and $\alpha \in [0,1]$ models the drag on the vehicle. Note that if the applied force on the vehicle is zero, the velocity decreases by the factor $(1-\alpha)$ every time index.

a. Assuming the vehicle starts at the origin at rest, i.e. $p(1) = (0,0)$ and $v(1) = (0,0)$, where we take $k = 1$ as the initial time index to simplify indexing. The problem is to find $f(1)$, $f(2)$, $f(3)$, …$f(K)$ (all$\in \mathbf{R}^2$) that minimize the cost function

$$J = \sum_{k=1}^{K} \left\| f(k) \right\|^2$$

subject to the constraints

$$p(k_i) = w_i \qquad i = 1,2,...M$$

where k_i are integers between 1 and K. The constraints mean that the vehicle is required to go through the locations w_i at instants k_i while there is absolutely no requirement on the velocity at the specified points. Solve for T = 0.1, m = 1, α = 0.1, K = 100 and M =4. The four constraints are

$$p(10) = (2,2)$$
$$p(30) = (-2,3)$$
$$p(40) = (4,-3)$$
$$p(80) = (-4,-2)$$

- Plot the x and y components of the force as a function of time.
- Plot the trajectory of the vehicle over time in the xy plane (use the MATLAB 'scatter' command).

b. Repeat all the requirements in part (a) for the same constraints but with different initial conditions $p(1) = (-4,-2)$ and $v(1) = (0,0)$.

1. Your MATLAB file should start with the following four lines
   % Name:
   % Section:
   % Seat Number:
   clc;clear all;close all;

2. In your code, you should make very clear the matrices **A** and **B** of the SS system. Also, you should make very clear the controllability matrix (or matrices) you will calculate and how you obtain them. If you will calculate more than one controllability matrix, be sure to give the variable name **C**_k where k is the time index associated with the controllability matrix.

3. Your output should produce in the command window: the matrices **A** and **B**, the values of the minimum energy in each case (a) and (b), and four figures showing the force components and the trajectory in both cases (a) and (b).

4. You should submit only the m-file as an attachment in an **empty email** to controlsysproject3rdcomm2016@gmail.com. Your email subject should be **SeatNo_xxx**. The deadline to submit the project is Wednesday May 4th no later than 8.00 am. Any projects submitted after that will simply be deleted.