Control Systems And Their Components (EE391)

Lec. 1: Course Introduction and System Modeling

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Basic Course Inforrmation

- Textbooks
  - "Modern Control Systems," R. Dorf and R. Bishop, Prentice Hall, 12th ed.
  - "Automatic Control Systems," F. Golnaraghi and B. Kuo, John Wiley & Sons. 9th ed.
  - "Linear State-Space Control Systems," R. Williams and D. Lawrence, John Wiley & Sons.
- Other Supplementary Material
  - Course Notes at MIT open courseware
    - "Feedback Control Systems", http://ocw.mit.edu/courses/aeronautics-and-astronautics/16-30-feedback-control-systems-fall-2010/
- Prerequisites
  - Signals and Systems
  - Linear Algebra
    - Excellent background refresher are the video lectures of Prof. Gilbert Strang at MIT http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/video-lectures/

Basic Course Inforrmation

- Computer Tools
  - MATLAB with control system toolbox

- Grading (150 marks)
  - Final (90 marks)
  - Year work (60 marks)
    - Midterm
    - Attendance
    - Quizzes
    - Project

- Course Webpage

  http://eng.staff.alexu.edu.eg/~mosman/

Course Outline

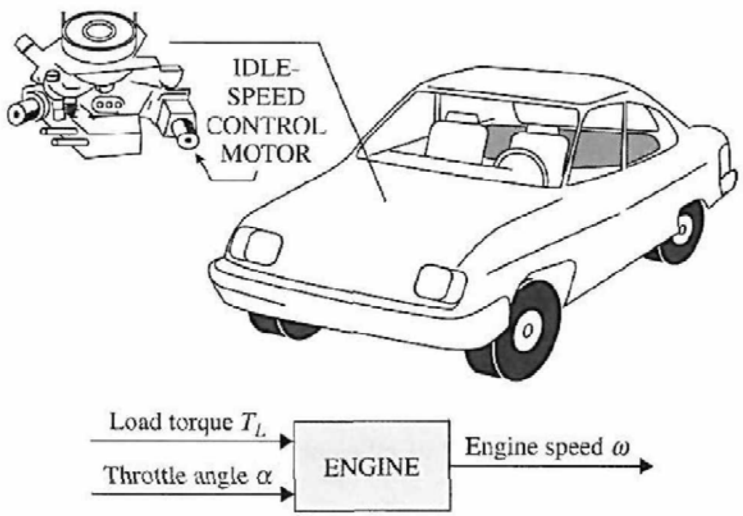
- Introduction to Control Systems
- Mathematical Modeling of Systems
- Time Domain Analysis of Feedback Control Systems
- Root Locus Design Method
- Frequency Domain Analysis of Feedback Control Systems
- Frequency Response Design Methods
- State Space Modeling
- Design of Control Systems using State Space Approach

Lecture Outline

- Introduction
- Examples of Control Systems
- Control System Components and Their Terminology
- Classification of Control Systems
- Modeling of Systems (Obtaining differential equations for mechanical and electrical systems)
- Obtaining Transfer Functions from Differential Equations

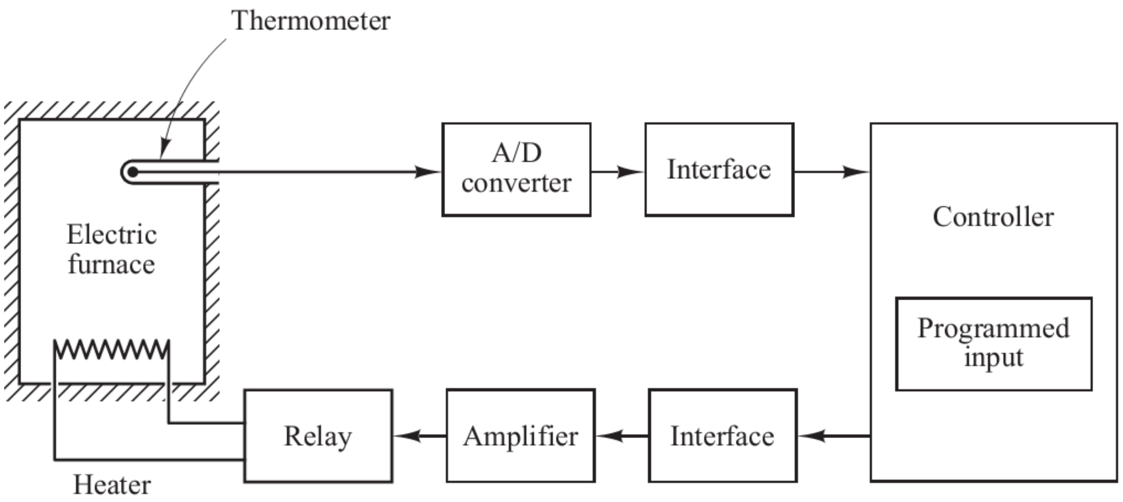
Introduction

- A <u>Control System</u> is a combination of components that act together to provide a desired response or output
- Where it is used? Give examples
  - Idle speed controller of a car engine



Introduction

- Temperature Control of an electric furnace



- Robotic arms in a production line of a car (watch this video)

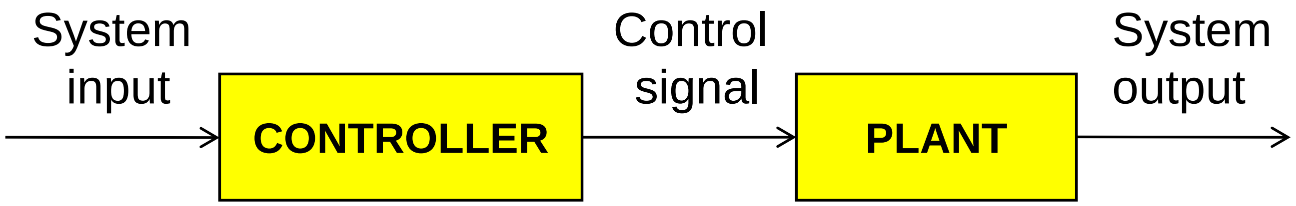
Basic Components and Terminology

Plant	It is the plant, process, or system under control
Controlled variable	The quantity or variable that is controlled by the control system and is generally the system output
Reference input	It is the reference or desired value of the controlled variable often provided as an input to the control system
Controller	It is the device that generates the required control signal
Actuator	It is the device that causes the plant or the process to provide the desired output by taking a form of energy (e.g. electric, hydraulic …) and producing motion (mechanical energy)
Sensor	The device that measures the output or controlled variable to be used by the controller

Classification of Control Systems

Open Loop vs. Closed Loop

- An open loop control system does not employ feedback, i.e. the control signal is independent of the output

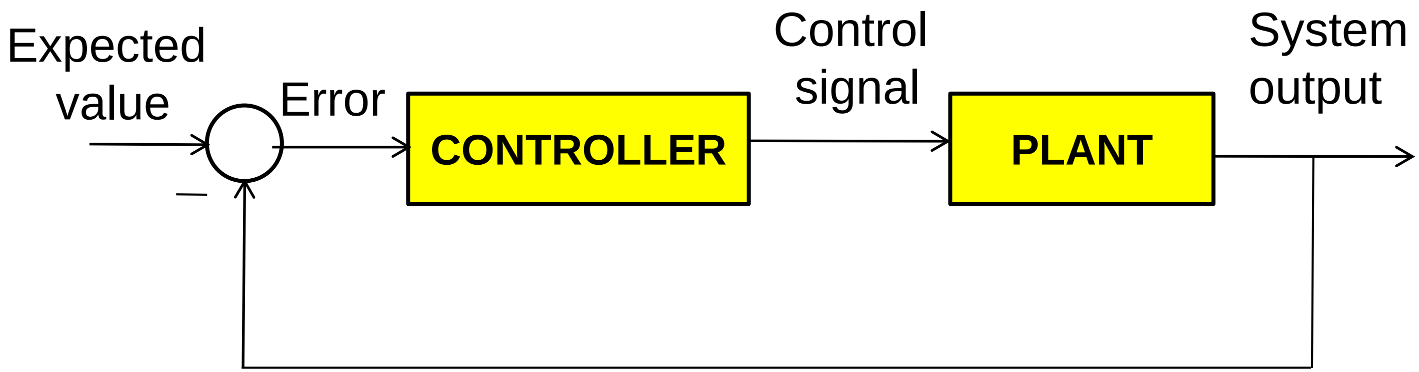


- Requires calibration to have different control signals preset for different reference inputs
- Requires exact perfect knowledge of system
- Simpler than closed loop but will not perform well in presence of disturbances

Classification of Control Systems

Open Loop vs. Closed Loop

- A closed loop control system uses feedback, i.e. the control signal depends on the output (measured by sensor)



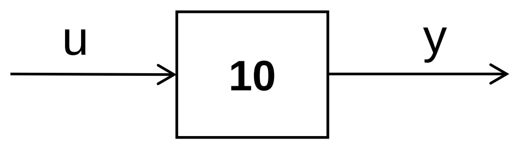
- Negative feedback: Compare the output with the desired or expected value and take action actions based on difference
- Feedback is very important in the field of control theory!!
- Why Feedback?? Does it have advantages?? (watch this video)

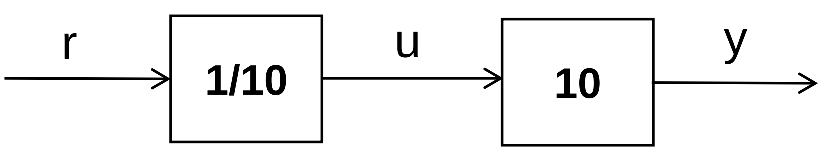
Classification of Control Systems

Advantages of Feedback

- Less impacted by variations of system model (more robust)

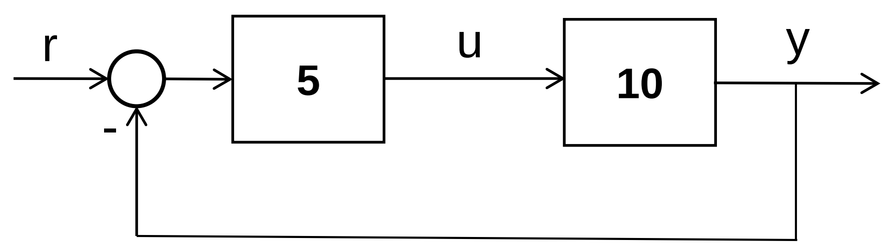
  Example: say we want y to track a certain reference input r





Open loop

y/r = 1

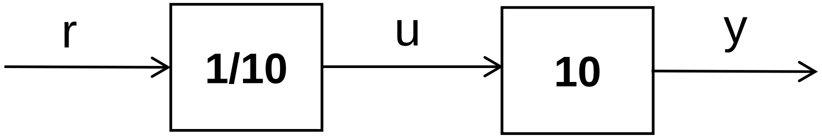


Closed loop

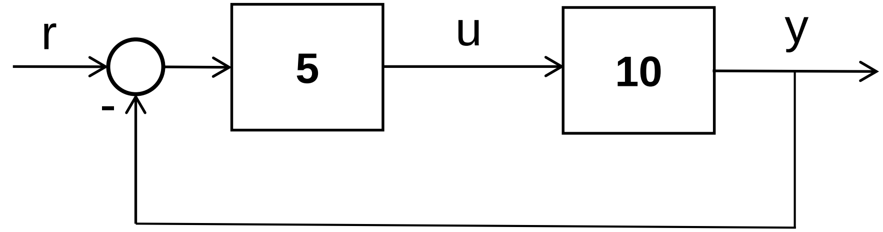
y/r = 50/51 = 0.98

Classification of Control Systems

Advantages of Feedback

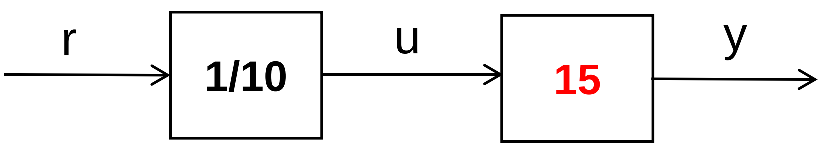


y/r = 1

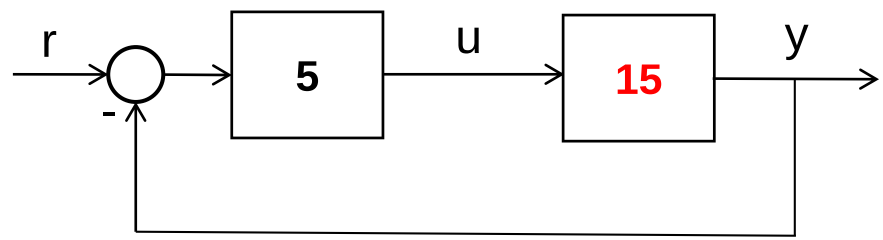


y/r = 50/51 = 0.98

If plant gain changes from 10 to 15



y/r = 1.5



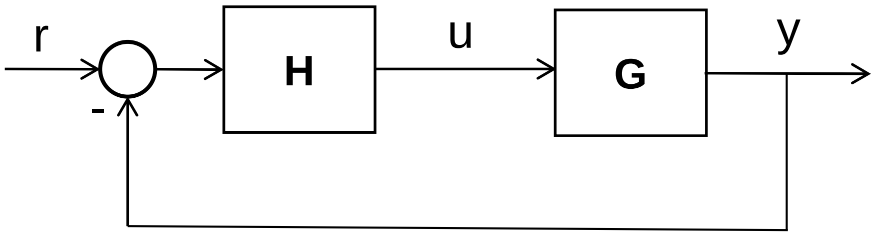
y/r = 75/76 = 0.9868

Closed loop less sensitive to change in plant gain

Classification of Control Systems

Advantages of Feedback

Generally speaking



$$T = \frac{y}{r} = \frac{GH}{1 + GH}$$

When open loop transfer func G changes by ΔG, closed loop transfer func *T* changes by Δ*T*

$$\Delta T = \frac{\partial T}{\partial G} \Delta G$$

$$= \frac{(1 + GH)H - GH^2}{(1 + GH)^2} \Delta G = \frac{H}{(1 + GH)^2} \Delta G$$

$$= \frac{1}{(1 + GH)} \frac{GH}{(1 + GH)} \frac{\Delta G}{G}$$

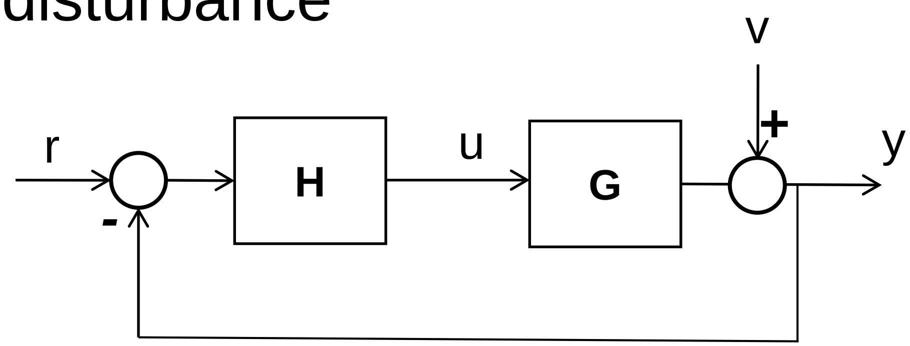
$$\therefore \frac{\Delta T / T}{\Delta G / G} = \frac{1}{(1 + GH)}$$

Sensitivity to change in open loop gain is reduced by feedback

Classification of Control Systems

Advantages of Feedback

- Less impacted by external disturbance
  - By superposition, we write output due to input and disturbance as



$$y = r\frac{GH}{1+GH} + v\frac{1}{1+GH}$$

  - Disturbance is clearly suppressed by feedback

Classification of Control Systems

Advantages of Feedback

- Less impacted by variations of system model
- Less impacted by external disturbance
- Reduce nonlinearity of system
- Increased bandwidth

Cost of Feedback

- Complexity
- Potential instability
- Reduced gain

Classification of Control Systems

Linear vs Nonlinear

- Linear systems satisfy the properties of superposition and homogeneity
- A system described by the i/o relationship *f* is linear if

$$y_1 = f(x_1), y_2 = f(x_2)$$

$$f(\alpha_1 x_1 + \alpha_2 x_2) = \alpha_1 f(x_1) + \alpha_2 f(x_2) = \alpha_1 y_1 + \alpha_2 y_2$$

- System whose dynamic response is described by linear differential equation is linear

  example: $a_2\ddot{y} + a_1\dot{y} + a_0 y = b_2\ddot{x} + b_1\dot{x} + b_0 x$

- System described by nonlinear differential equation is nonlinear

  example $a_2\ddot{y} + a_1\dot{y} + a_0 y = x^2 + sin(x)$

Classification of Control Systems

Time variant vs Time invariant

- Time invariant systems have parameters that are stationary with respect to time
- A system described by the i/o relationship f is time invariant if

If $$y(t) = f\big(x(t)\big)$$

Then $$f\big(x(t-\Delta)\big) = y(t-\Delta)$$

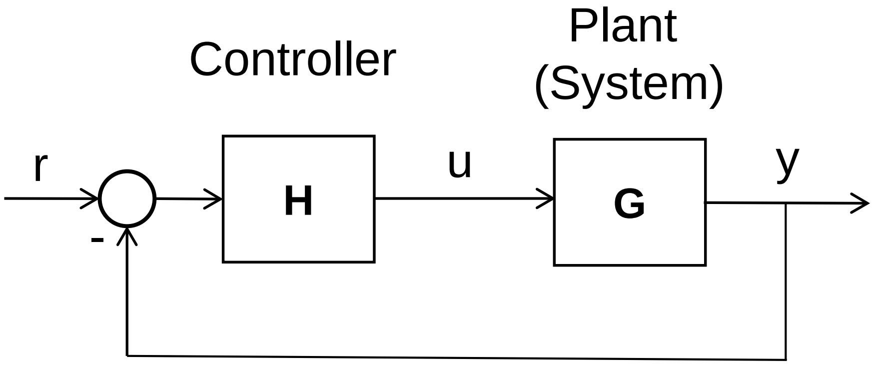
Basic requirements of a control system

- **Stability**: refer to the ability of a system to recover equilibrium
- **Quickness**: refer to the duration of transient process before the control system to reach its equilibrium
- **Accuracy**: refer to the size of steady-state error when the transient process ends

  (Steady state error = desired output – actual output)

- Above three performance specifications sometime contradict so we need to make compromises
- In the course we will study both **Analysis** & **Design**

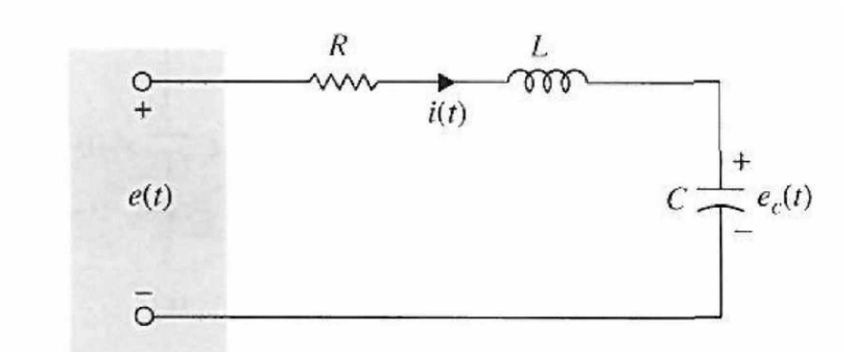
Modeling of Physical Systems

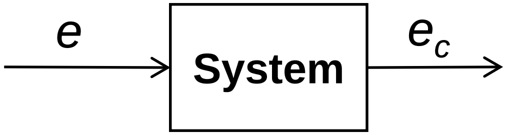


- Before designing a controller that will achieve certain objectives of the overall closed loop system, an accurate mathematical model describing the behavior of the plant or system under control is a key step (Find G first)
- A <u>mathematical model</u> is a mathematical description of the dynamic relationship between the input and output of the system (usually it is a diff. Eq.)
- Systems can have components that are mechanical, electrical, fluid, thermal,…. (focus on both mechanical and electrical)

Modeling of Electrical Systems

- Use Kirchoff's law
- Example: Assume we have an RLC circuit where the input and output are as shown





$$e(t) = Ri(t) + L\frac{di}{dt} + e_c(t) \quad \Leftarrow \quad i(t) = C\frac{de_c(t)}{dt}$$

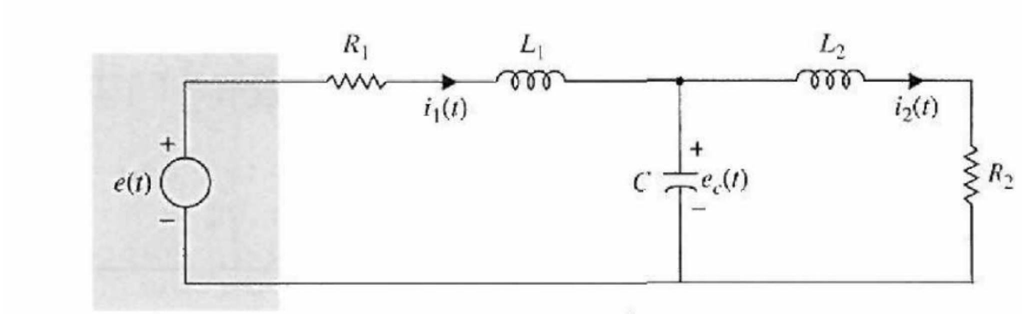
$$e(t) = RC\frac{de_c(t)}{dt} + LC\frac{d^2e_c(t)}{dt^2} + e_c(t)$$

$$\boxed{LC\ddot{e}_c + RC\dot{e}_c + e_c = e} \quad \Leftarrow$$

i/o relationship
2^{nd} order linear ordinary differential equation with constant coefficients

Modeling of Electrical Systems

- Problem
  - Find the i/o differential equation that relates the input e(t) and output $e_c(t)$ in the following circuit. What will be the order of the diff. Eq.? (Hint: Use Laplace transform to convert diff. Equations into simple algebraic equations)



  - (Important Note: Order of Diff. Eq. = number of energy storage elements in circuit)

Modeling of Mechanical Systems

- Mechanical systems involve either **translational**, **rotational** motions or a combination of both
- Translational motion takes place in a straight or a curved path, whereas rotational motion takes place around a fixed axis
- For translational motion, use Newton's law of motion

$$\sum forces = ma$$

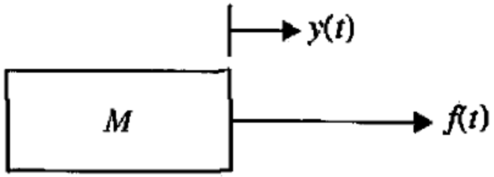
In the same direction

- Examples:
  - Single mass under external force

$$M \frac{d^2 y(t)}{dt^2} = f(t)$$

$$\boxed{M\ddot{y} = f}$$

y: displacement
$\dot{y}$: velocity
$\ddot{y}$: acceleration

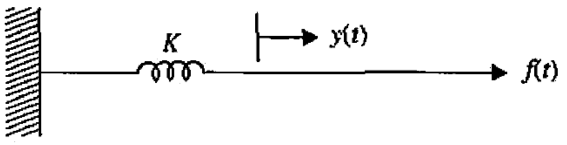


Modeling of Mechanical Systems

- Examples:
  - Linear spring: Force applied on spring is directly proportional to the displacement or deformation in the spring

$$Ky(t) = f(t)$$

$$\boxed{Ky = f}$$

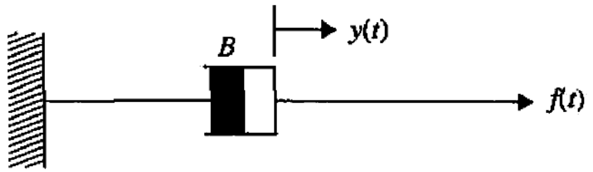


K: spring constant (N/m)

  - Viscous friction damper: produces a retarding force directly proportional to the velocity (used to model friction force acting on a translating mass)

$$B\frac{dy(t)}{dt} = f(t)$$

$$\boxed{B\dot{y} = f}$$



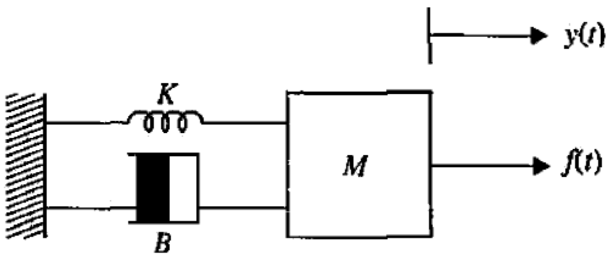
B: viscous frictional coefficient (N.s/m)

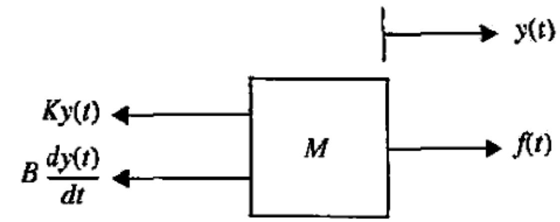
Other more complicated types of friction such as Coulomb friction exist but mostly when moving mechanical parts are well lubricated the viscous friction model is a good approximation

Modeling of Mechanical Systems

- Examples:
  - Mass-Spring-Damper system

Free Body Diagram (FBD)



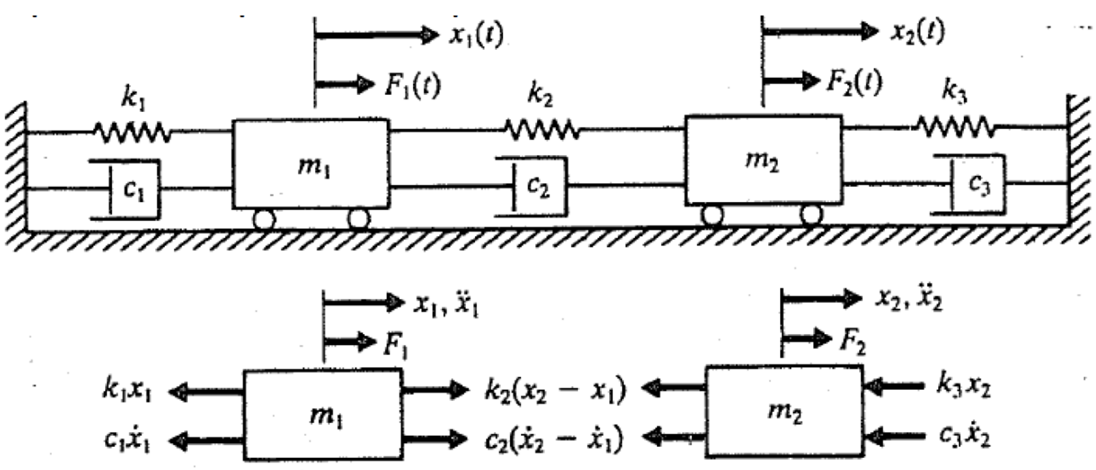


$$M\ddot{y} + B\dot{y} + Ky = f$$

i/o relationship
2nd order linear ordinary differential equation with constant coefficients

  - Homework problem

Solution will involve two coupled linear second order differential equations in x_1 and x_2



Modeling of Mechanical Systems

- For rotational motion, use Newton’s law of motion again

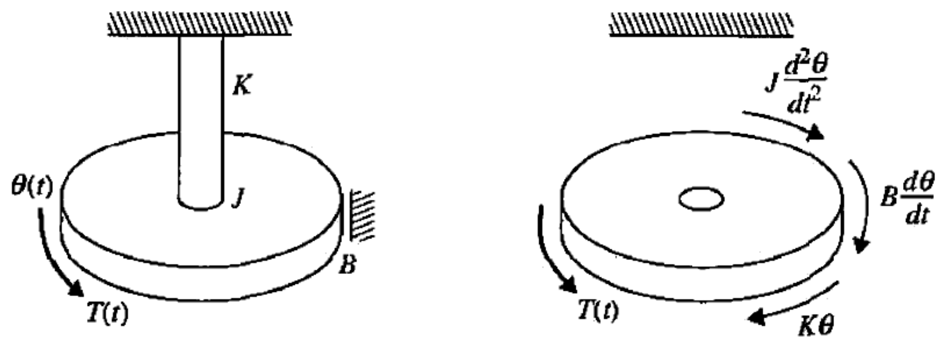
$$\sum Torques = J\ddot{\theta}$$

J: moment of inertia of rotating object calculated around the axis of rotation (kg.m^2)
$\ddot{\theta}$: Angular acceleration (rad/s^2)

- Examples:
  - Disc under applied torque + rotational spring + friction

$$\boxed{J\ddot{\theta} + B\dot{\theta} + K\theta = T}$$

B and *K* are also the spring and and viscous friction coefficients but have different units compared to translational motion case



How to calculate *J* ???

Modeling of Mechanical Systems

- For a rotating disc with mass *M* and radius *R*, prove that

$$J = \frac{1}{2} M R^2$$

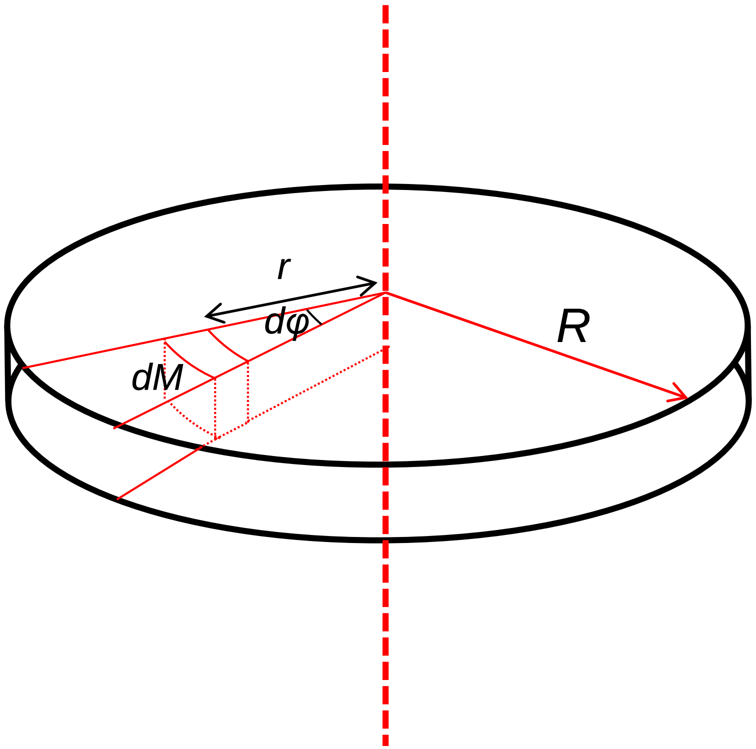
$$dJ = dM r^2$$

$$dM = M \frac{dr \cdot r d\varphi}{\pi R^2}$$

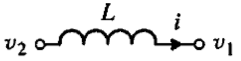
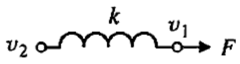
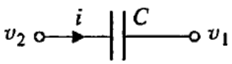
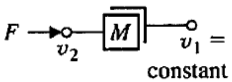
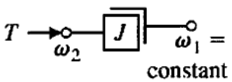
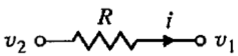
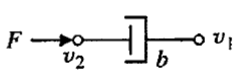
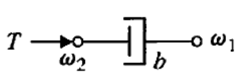
$$dJ = M \frac{r^3 dr d\varphi}{\pi R^2}$$

$$J = \frac{M}{\pi R^2} \int_0^R r^3 dr \int_0^{2\pi} d\varphi$$

$$J = \frac{1}{2} M R^2$$



Analogy between electrical and mechanical

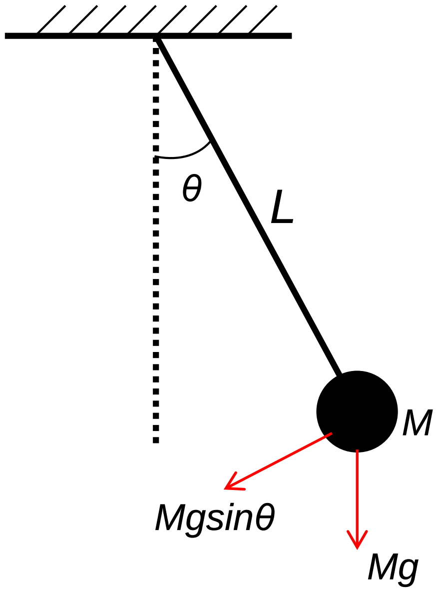
Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}Cv_{21}{}^2$	
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}Mv_2{}^2$	
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J\omega_2{}^2$	
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}{}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}{}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}{}^2$	

Nonlinear system

- Example: Pendulum oscillator

$$\sum Torques = J\ddot{\theta}$$

$$MgLsin\theta = J\ddot{\theta}$$
$$MgLsin\theta = ML^2\ddot{\theta}$$



$$ML\ddot{\theta} - Mgsin\theta = 0$$

$$\ddot{\theta} - \frac{g}{L}sin\theta = 0$$ ⇦ Nonlinear

<u>But for small θ</u> $sin\theta \approx \theta$

$$\ddot{\theta} - \frac{g}{L}\theta = 0$$ ⇦ After linearization

What is the formal way that we can use to linearize any model around the equilibrium point ??? Taylor series